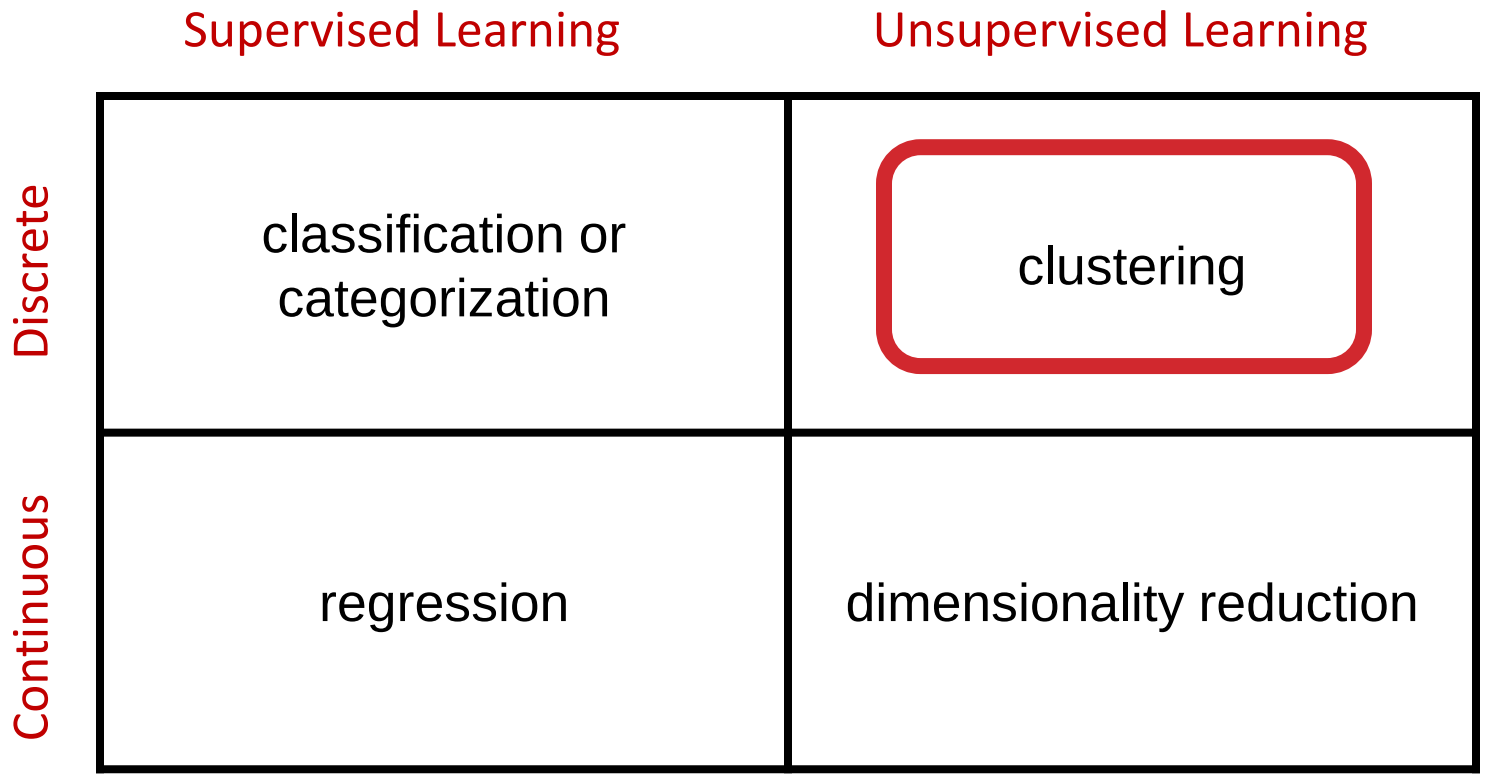
A PRACTICAL GUIDE TO MACHINE LEARNING

MACHINE LEARNING PROBLEMS

(Boosted-) Decision Trees

K-Means
Agglomerative clustering
DBScan



(Boosted-) Decision Trees

PCA

CLUSTERING STRATEGIES

K-means

- Iteratively re-assign points to the nearest cluster center

Agglomerative clustering

- Start with each point as its own cluster and iteratively merge the closest clusters

Mean-shift clustering

- Estimate modes of PDF (i.e., the value x at which its probability mass function takes its maximum value)

Spectral clustering

- Split the nodes in a graph based on assigned links with similarity weights

DBSCAN (Density-based spatial clustering of applications with noise)

Mixture Gaussian clustering

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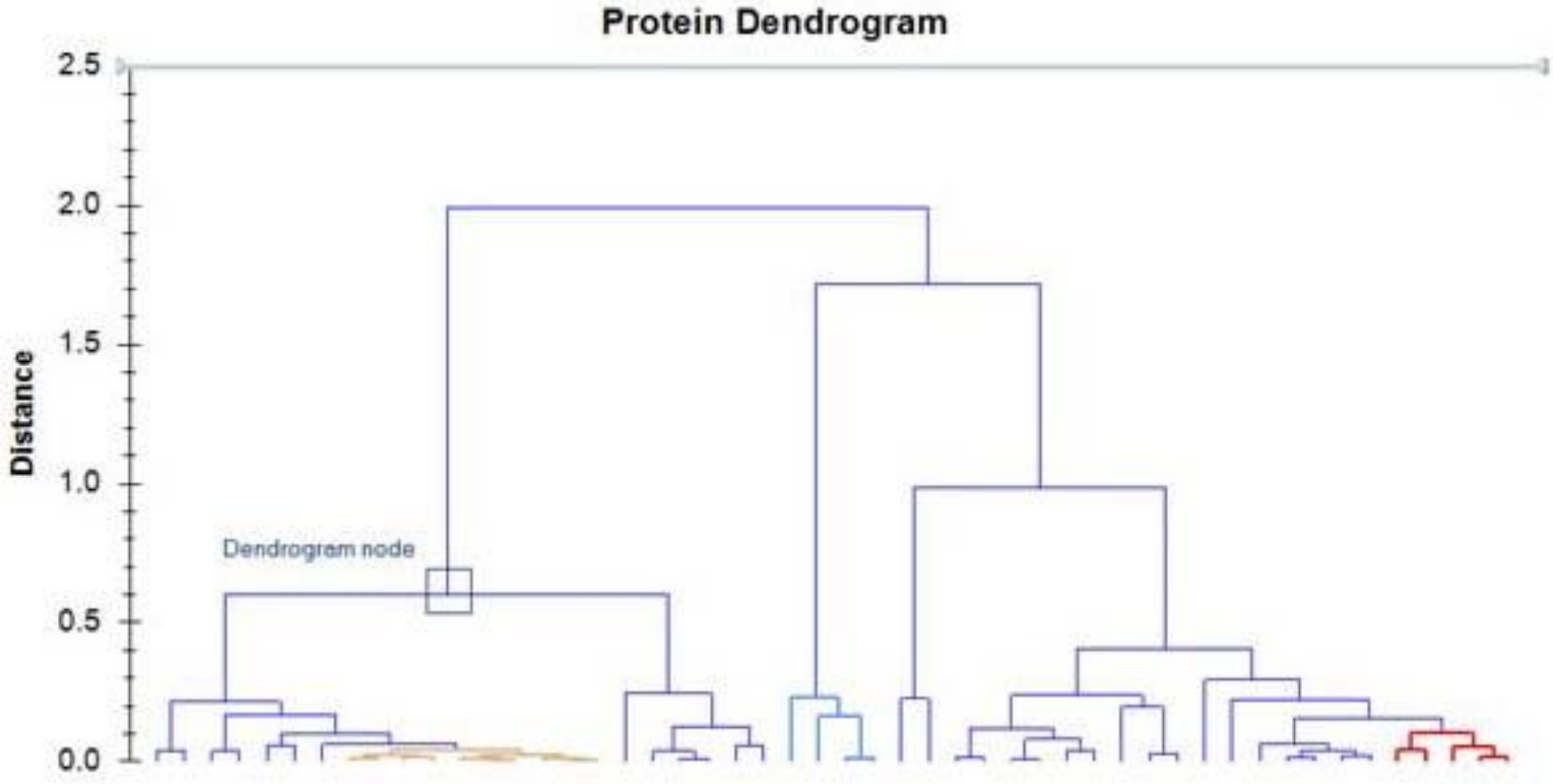
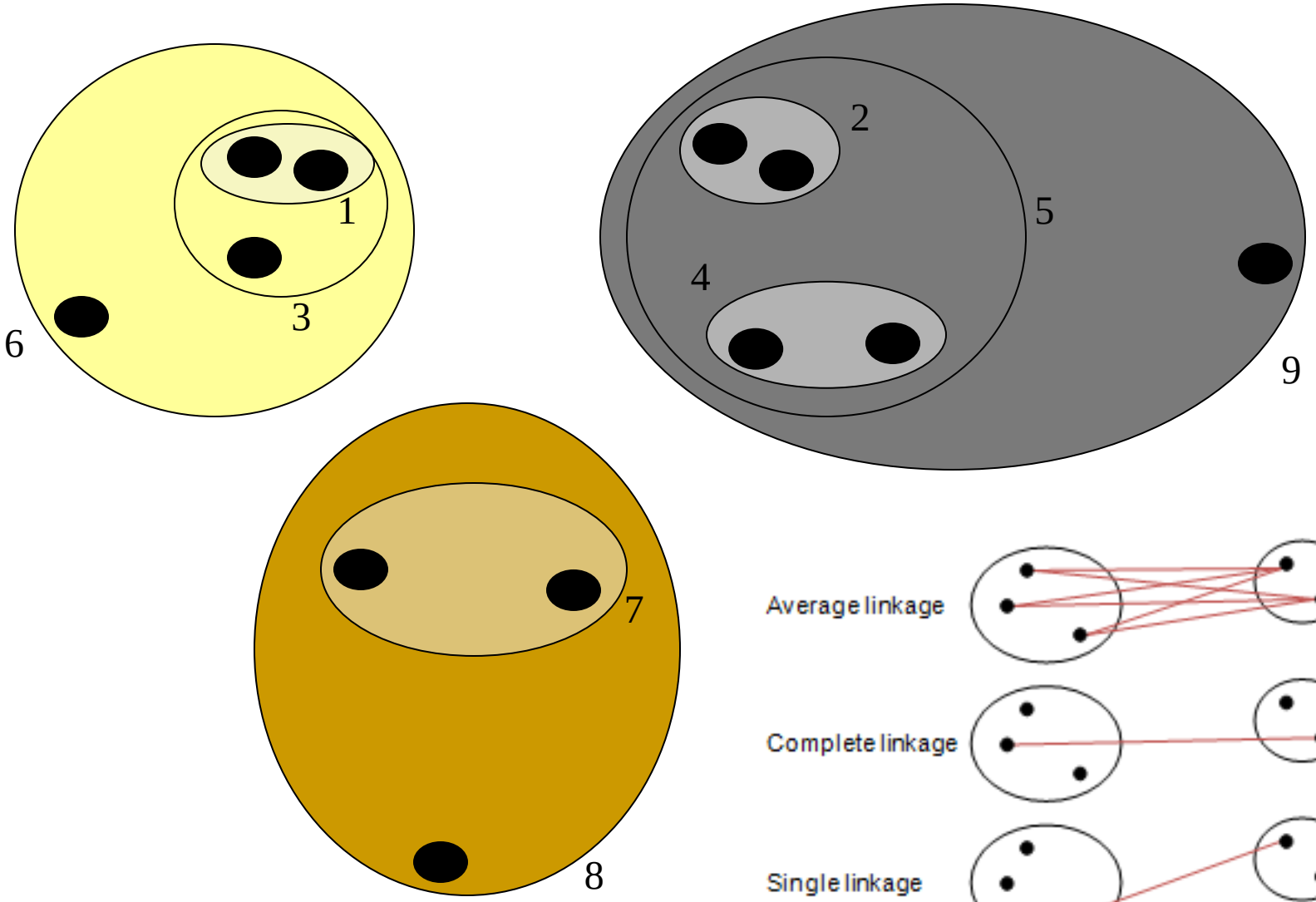
DBSCAN (Density-based spatial clustering of applications with noise)

As we go down this chart, the clustering strategies have more tendency to transitively group points even if they are not nearby in feature space

DENDROGRAM EXAMPLE



Group Agglomerative Clustering



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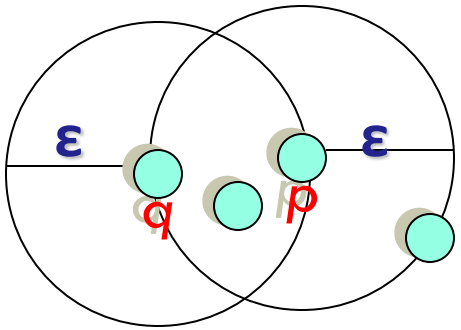
ε-NEIGHBORHOOD

ε-Neighborhood – Objects within a radius of ε from an object.

$$N_{\varepsilon}(p):\{q \mid d(p,q) \leq \varepsilon\}$$

"High density" - ε-Neighborhood of an object contains at least *MinPts* of objects.



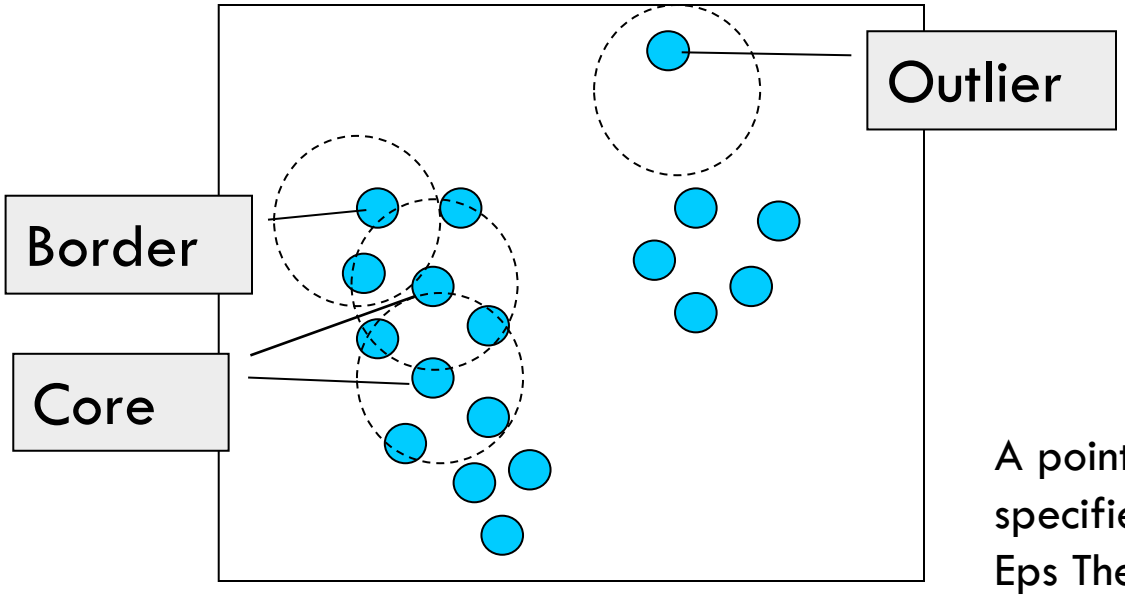
ε-Neighborhood of *p*

ε-Neighborhood of *q*

Density of p is "high" (MinPts = 4)

Density of q is "low" (MinPts = 4)

CORE, BORDER & OUTLIER



$\varepsilon = 1$unit, MinPts $= 5$

Given ε and *MinPts*, categorize the objects into three exclusive groups.

A point is a core point if it has more than a specified number of points (MinPts) within Eps These are points that are at the interior of a cluster.

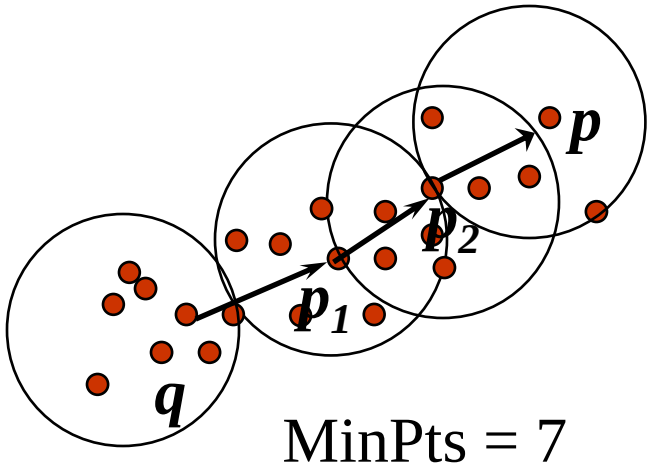
A border point has fewer than MinPts within Eps, but is in the neighborhood of a core point..

A noise point (outlier) is any point that is not a core point nor a border point.

DENSITY-REACHABILITY

Density-Reachable (directly and indirectly):

- A point p is directly density-reachable from p2;
- p2 is directly density-reachable from p1;
- p1 is directly density-reachable from q;
- p←p2←p1←q form a chain.



p is (indirectly) density-reachable from q

q is not density- reachable from p?

DBSCAN ALGORITHM

Input: The data set D

Parameter: ε, MinPts

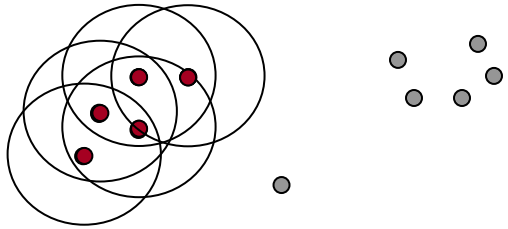
For each object p in D
    if p is a core object and not processed then
        C = retrieve all objects density-reachable from p
        mark all objects in C as processed
        report C as a cluster
    else mark p as outlier
    end if

End For

DBSCAN ALGORITHM: EXAMPLE

Parameter

- $\varepsilon = 2$ cm
- *MinPts* = 3



```
for each o ∈ D do
    if o is not yet classified then
        if o is a core-object then
            collect all objects density-reachable from o
            and assign them to a new cluster.
        else
            assign o to NOISE
```

DBSCAN ALGORITHM: EXAMPLE

Parameter

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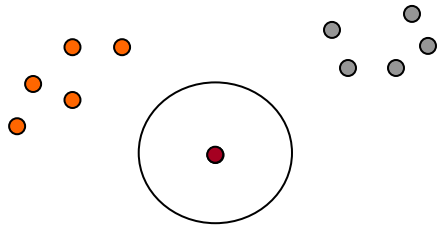

for each $o \in D$ **do**
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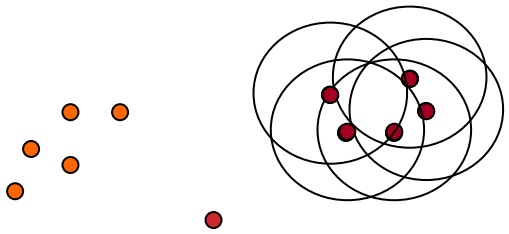

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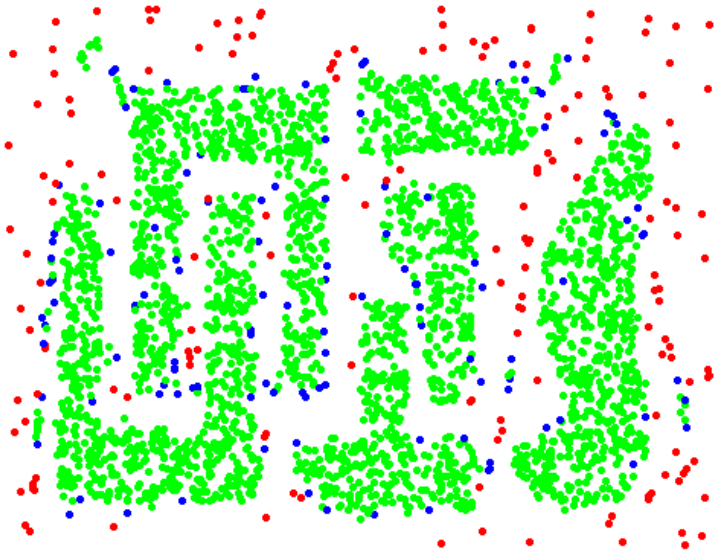
EXAMPLE

Original Points

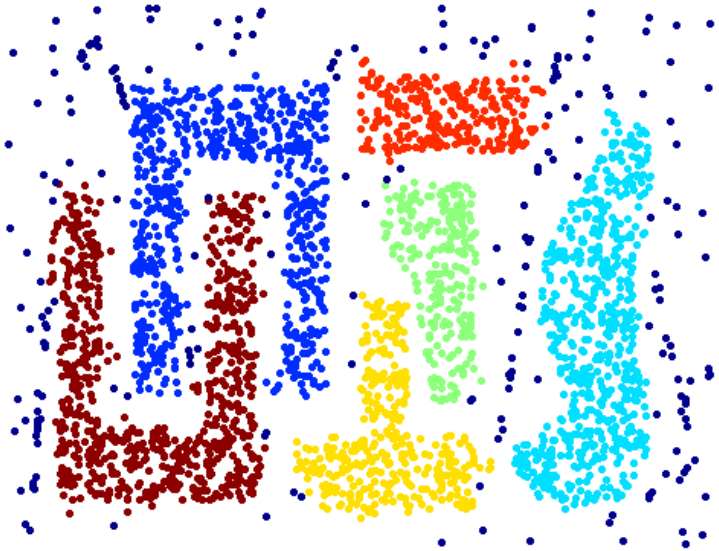


Point types: core, border and outliers

$\varepsilon = 10$, MinPts = 4

WHEN DBSCAN WORKS WELL

Original Points

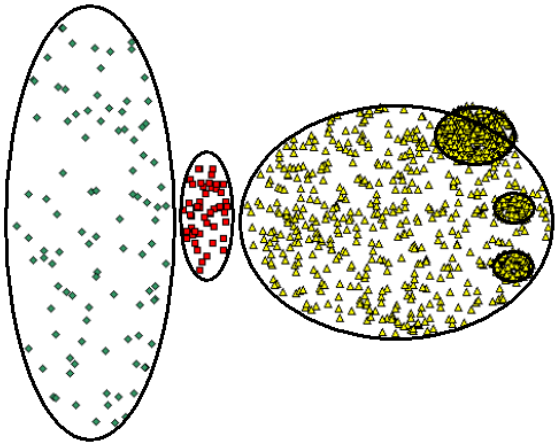


Clusters

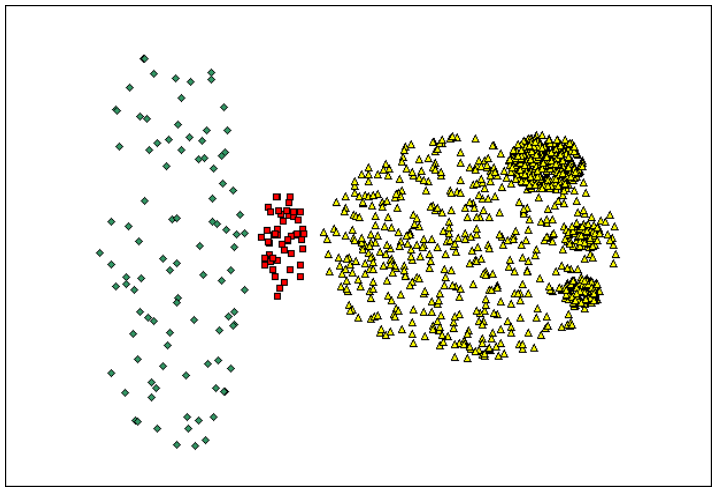
- Resistant to Noise
- Can handle clusters of different shapes and sizes

CAN YOU CREATE AN EXAMPLE FOR WHICH DBSCAN WILL NOT WORK WELL

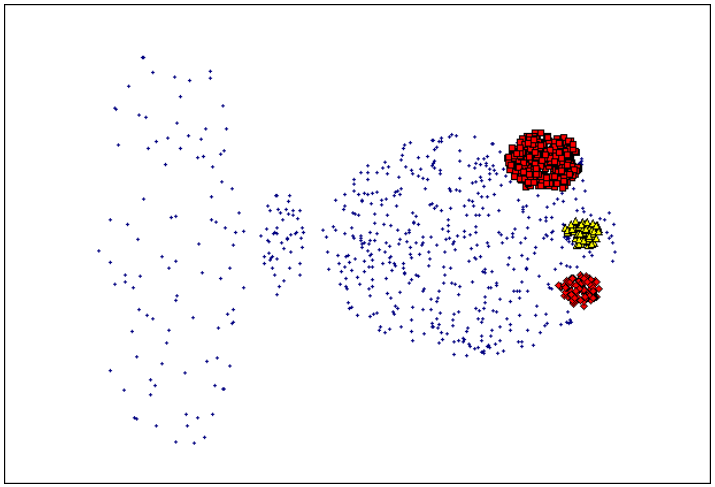
WHEN DBSCAN DOES NOT WORK WELL



Original Points



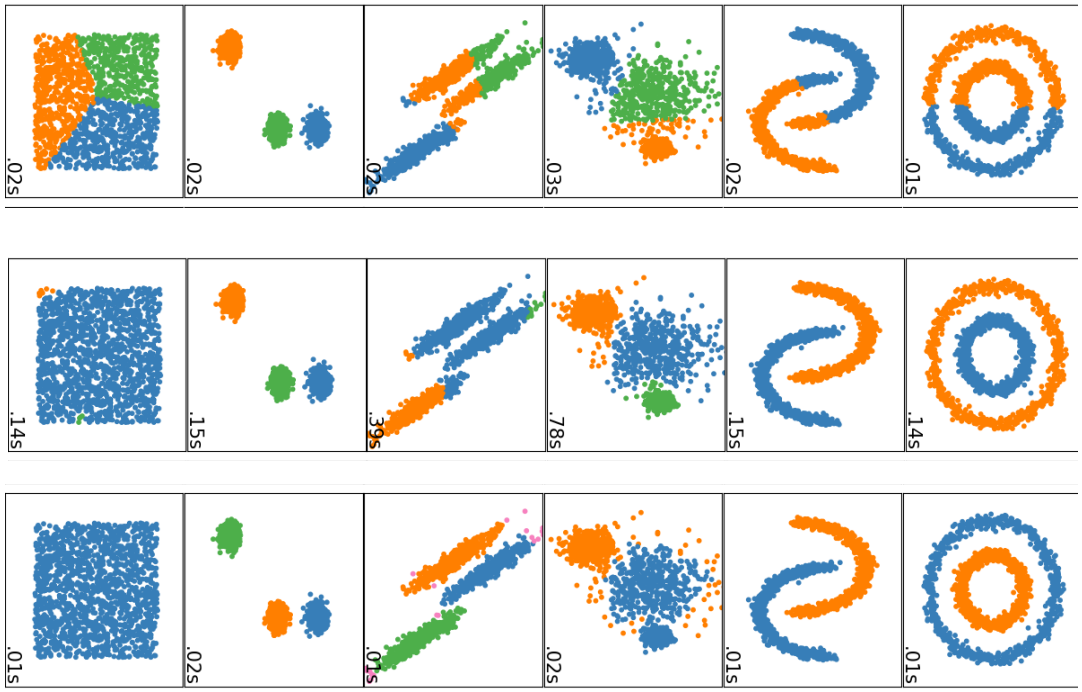
(MinPts=4, Eps=9.92).



(MinPts=4, Eps=9.75)

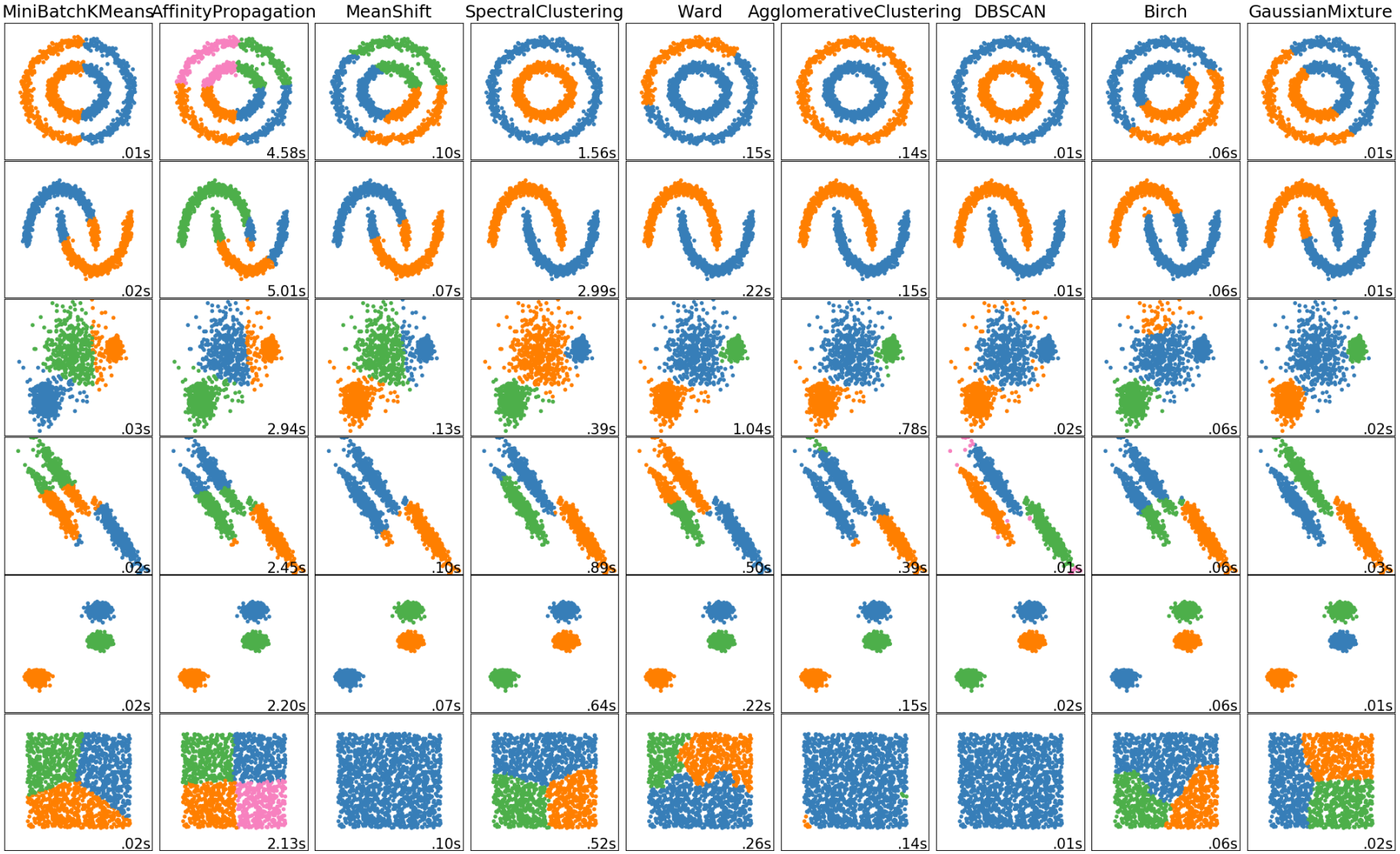
- Cannot handle Varying densities
- Sensitive to parameters

CLICKER - HTTPS://CLICKER.CSAIL.MIT.EDU/6.S079/



A	B	C
KMeans	Aggl. clustering	KMeans
DBScan	KMeans	Aggl. clustering
Aggl. clustering	DBScan	DBScan

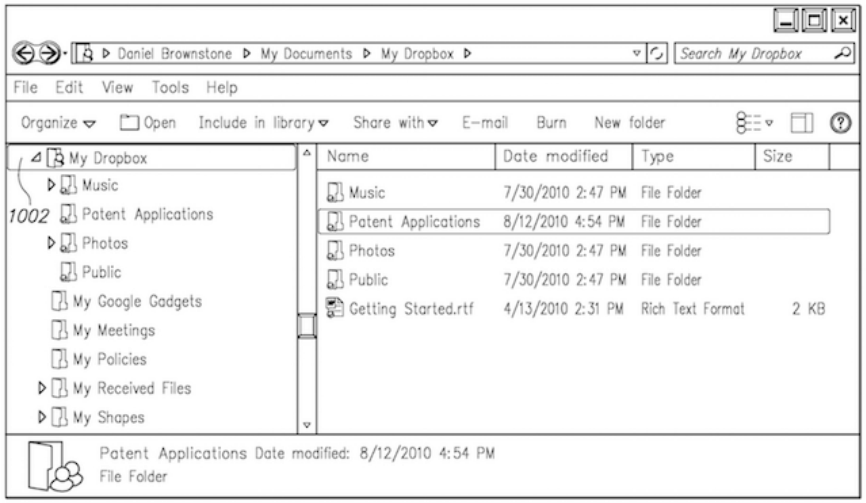
CLUSTERING


MiniBatchKMeans
AffinityPropagation
MeanShift
SpectralClustering
Ward
AgglomerativeClustering
DBSCAN
Birch
GaussianMixture
.01s
4.58s
.10s
1.56s
.15s
.14s
.01s
.06s
.01s
.02s
5.01s
.07s
2.99s
.22s
.15s
.01s
.06s
.01s
.03s
2.94s
.13s
.39s
1.04s
.78s
.02s
.06s
.02s
.02s
2.45s
.10s
.89s
.50s
.39s
.01s
.06s
.03s
.02s
2.20s
.07s
.64s
.22s
.15s
.02s
.06s
.01s
.02s
2.13s
.10s
.52s
.26s
.14s
.01s
.06s
.02s

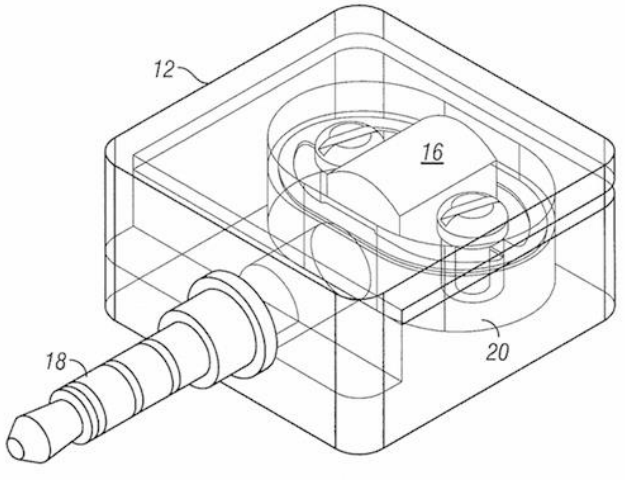
The EM Algorithm

Motivational Example

Around **300.000**
US **Patent** Applications
Granted **per Year**



Network folder synchronization (DropBox)



Systems and methods for decoding card s wipe signals (Square)

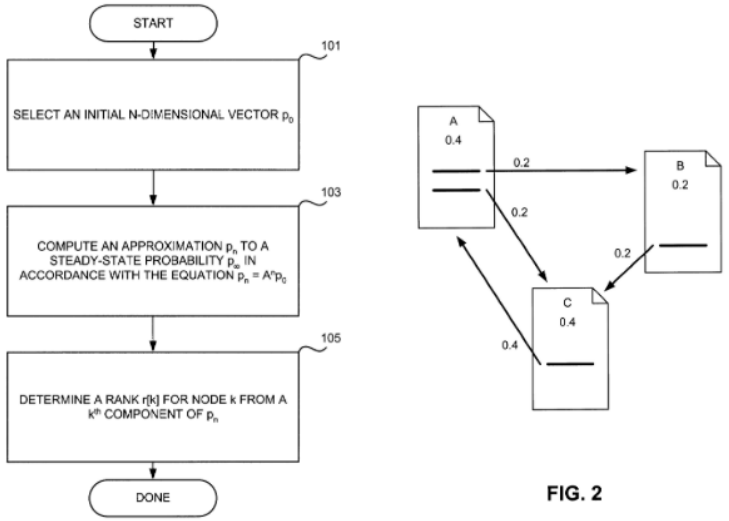


FIG. 2

Method for node ranking in a linked database (Google)

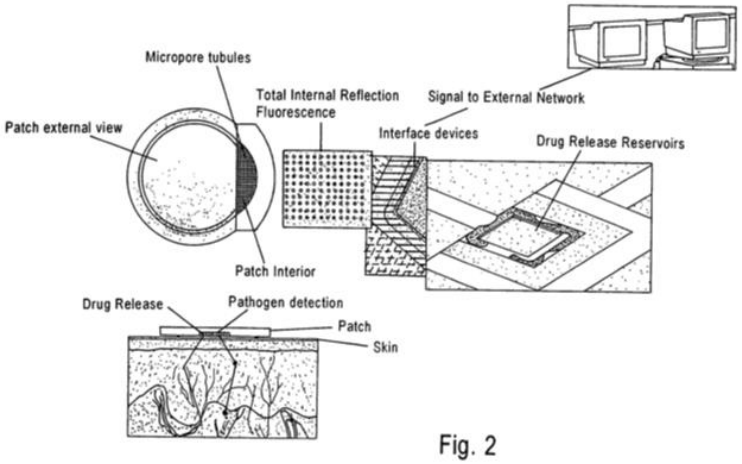
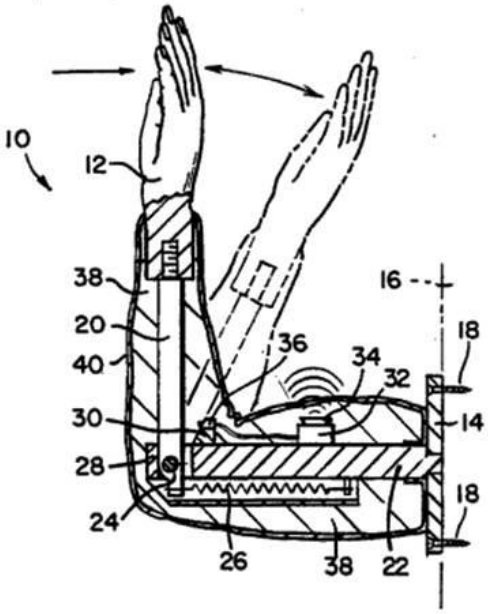


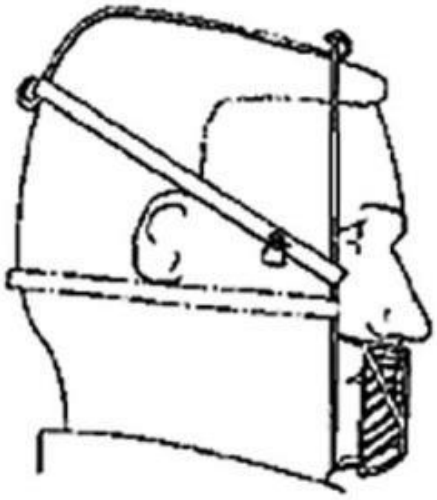
Fig. 2

Medical device for analyte monitoring and drug delivery (Theranos)

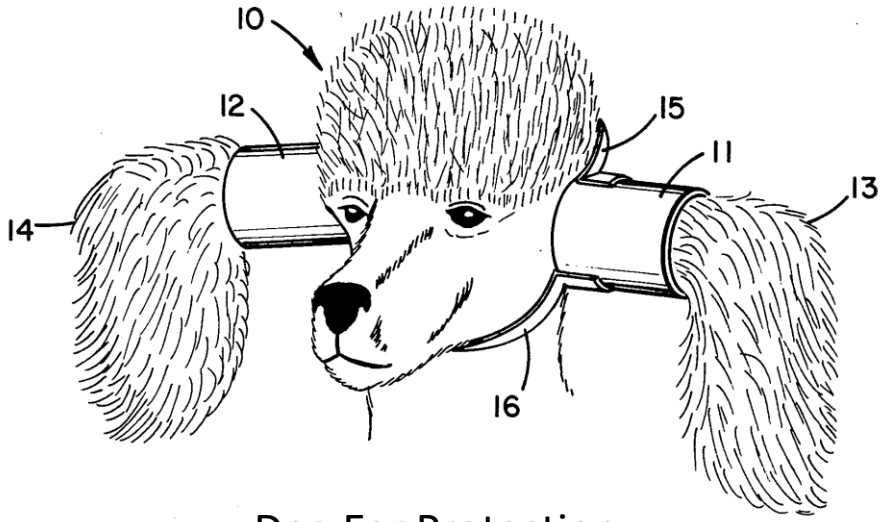


High-Five Machine

Gerbil Shirt



Anti Eating Device



Dog Ear Protection

Note, that I could not actually verify them as real patents, but they could easily be some

Goal:

We want to build a **model** to automatically **classify patents** into useful or bogus?

What do we need?

1. The patent data (easy thanks to Google Patents)
2. A training data set:
   some pre-labeled patents
3. A model

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How do we get a labeled data set?

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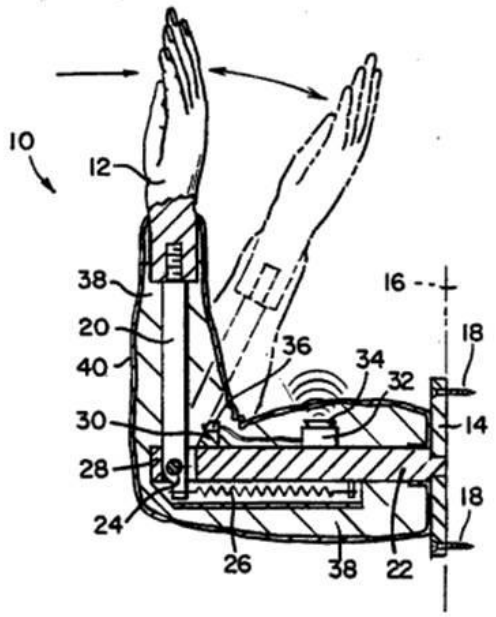
A Crowd Task

Is this Patent Bogus?

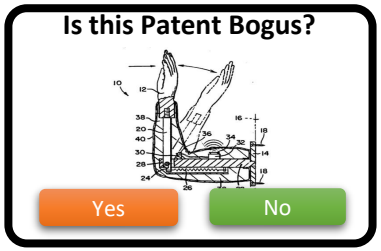


Yes

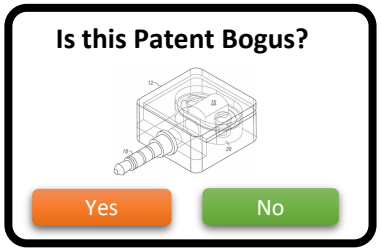
No

$l = 1$

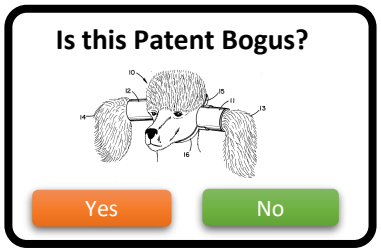
A Crowd Task



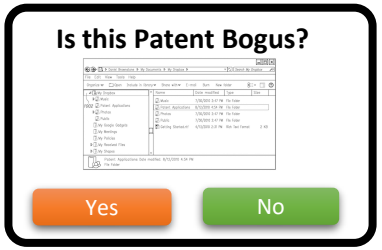
o_1



o_2



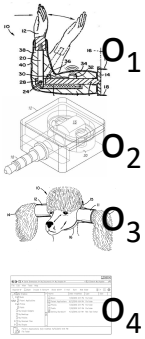
o_3



o_4

$\vdots$

$$l[n] =$$



o_1

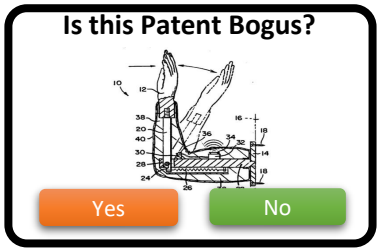
o_2

o_3

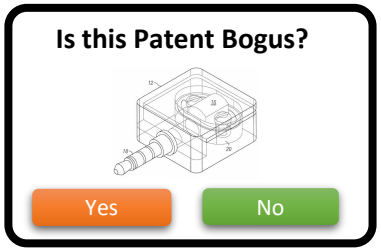
o_4

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}$$

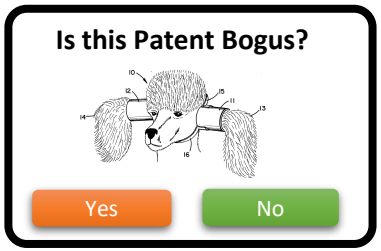
A Crowd Task



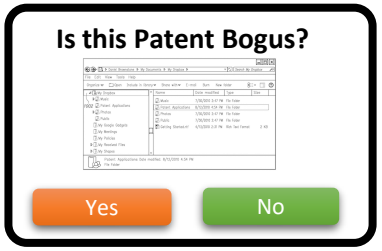
o_1



o_2



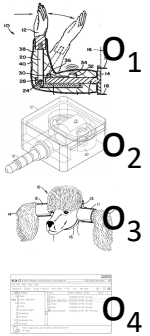
o_3



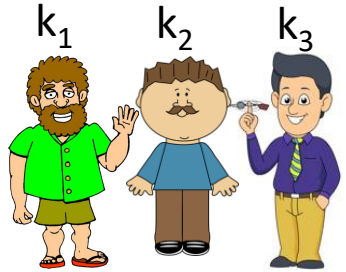
o_4

$\vdots$

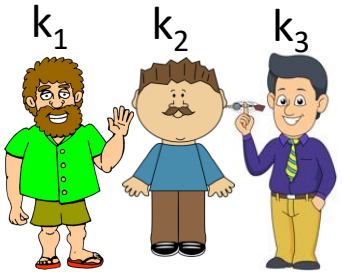
$$l[k][n] =$$



$$\begin{array}{c} o_1 \\ o_2 \\ o_3 \\ o_4 \\ \vdots \end{array} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ \vdots & \vdots & \vdots \end{pmatrix}$$



What should the final labels be?



$$l[k][n] = \begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \end{matrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ \vdots & \vdots & \vdots \end{pmatrix}$$

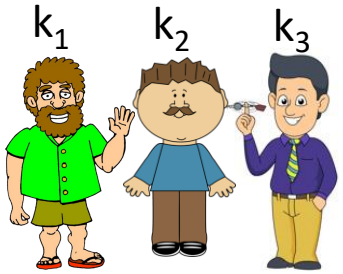
$$T(n) = \begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \end{matrix} \begin{pmatrix} ? \\ ? \\ ? \\ ? \\ \vdots \end{pmatrix}$$

Maximum Likelihood Estimate

- Given some data $X = (x_1, \ldots, x_n)$
- Model $\mathcal{L}(\theta, X) = p_\theta(X) = \prod_i^n p_\theta(x_i)$
- **Maximum Likelihood Estimator (MLE)**

$$\hat{\theta} = \underset{\theta \in \Theta}{\operatorname{argmax}}\, \mathcal{L}(\theta, X)$$

A Maximum Likelihood Estimate (MLE)



$$l[k][n] = \begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \end{matrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ \vdots & \vdots & \vdots \end{pmatrix}$$

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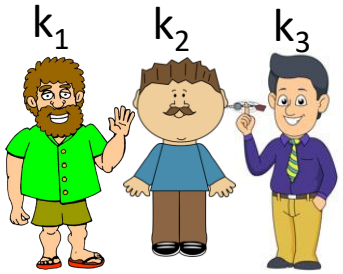
	Bogus	Not Bogus
	3/3	0/3
	0/3	3/3
	2/3	1/3
	1/3	2/3
	⋮	⋮

A Maximum Likelihood Estimate (MLE)



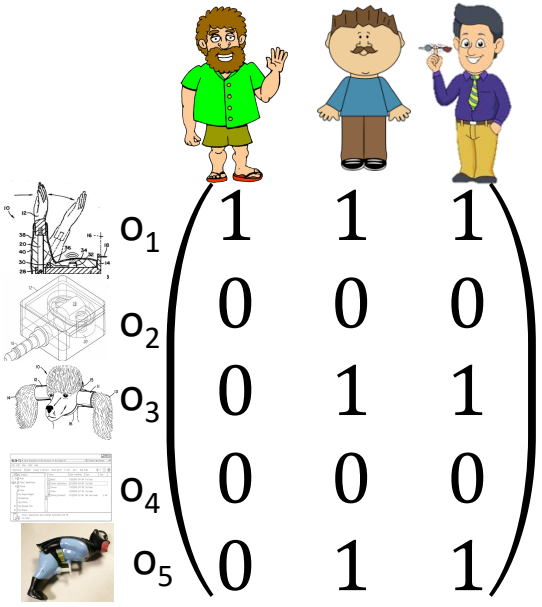
$$l[k][n] = \begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \end{matrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ \vdots & \vdots & \vdots \end{pmatrix}$$

What should the final labels be?

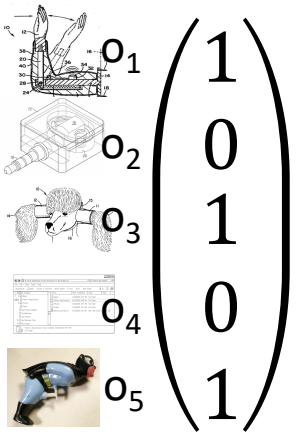
$$T(n) = \begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \end{matrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}$$

Bogus	Not Bogus
3/3	0/3
0/3	3/3
2/3	1/3
1/3	2/3
⋮	⋮

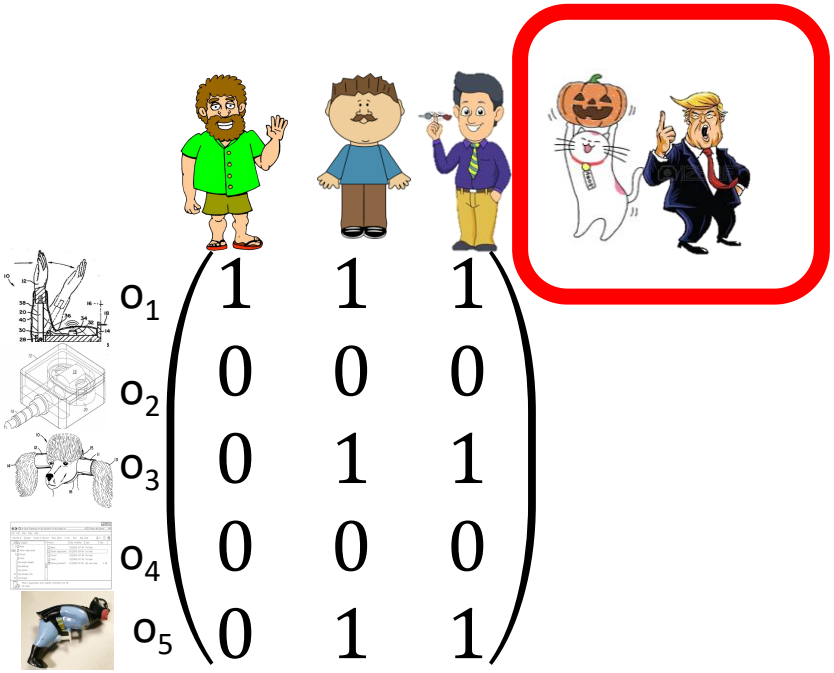
So Everything is Good



$$T(n) =$$

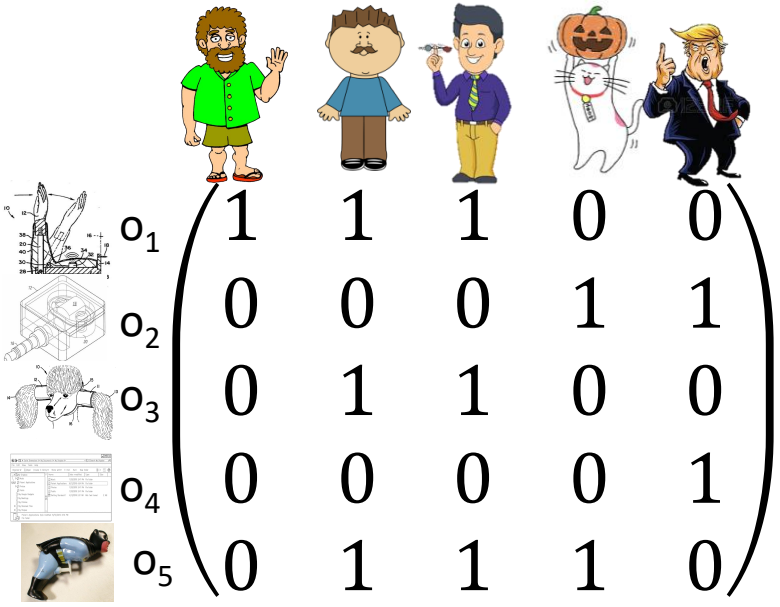


But what happens if we add **Crazy Cat with Pumpkin** and the **Trumpworker**

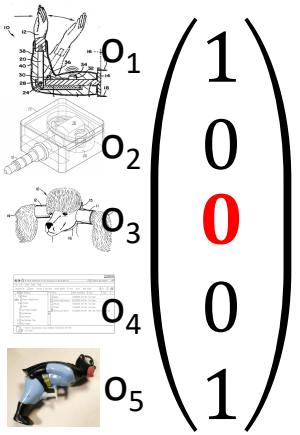


$$T(n) = \begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{matrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

What if the Workers do not have the same Quality?



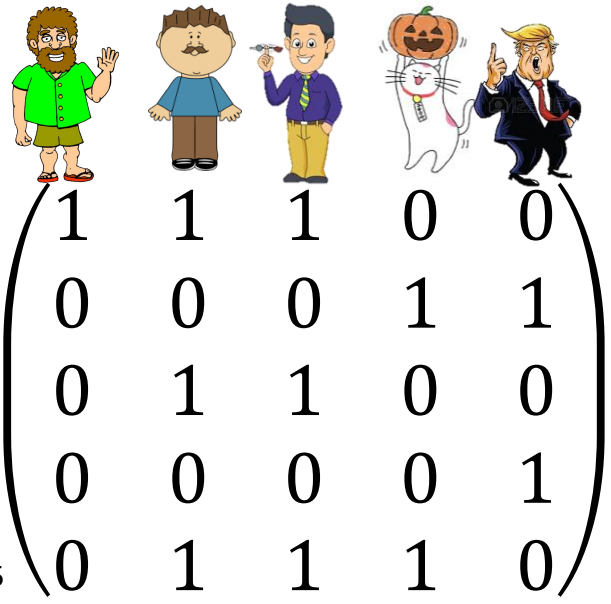
$$T(n) =$$



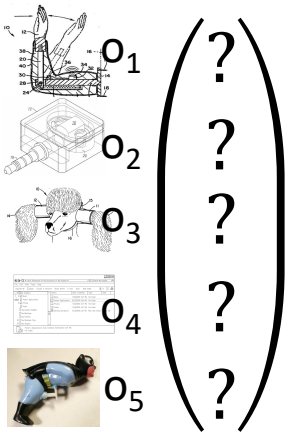
What if the Workers do not have the same Quality?

Latent (hidden) Variables

Z_1 Z_2 Z_3 Z_4 Z_5



$$T(n) =$$



Maximum Likelihood Estimate

- Given some data $X = (x_1, \ldots, x_n)$
- Model $\mathcal{L}(\theta, X) = p_\theta(X) = \prod_i^n p_\theta(x_i)$
- **Maximum Likelihood Estimator (MLE)**

$$\hat{\theta} = \underset{\theta \in \Theta}{\operatorname{argmax}}\, \mathcal{L}(\theta, X)$$

Maximum Likelihood Estimate

- Given some data X= $(x_1, \dots, x_n)$
- Model $\mathcal{L}(\theta, X, Z) = p_\theta(X, Z) = \prod_i^n p_\theta(x_i, z)$
- **Maximum Likelihood Estimator (MLE)**

$$\hat{\theta} = \underset{\theta}{\operatorname{argmax}}\, \mathcal{L}(\theta, X) = \sum_z p_\theta(X, Z)$$

  - Z has been marginalized
  - Hard to compute

Expectation Maximization Algorithm

Initialize $\theta \in \Theta$

For t = 0,1,2,…

E-Step: Calculate the expected value of the log likelihood function, with respect to the conditional distribution of Z given X under the current estimate of the parameters θ_t:

$$Q(Q|\theta_t) = E_{Z|X,\theta_t}[\log \mathcal{L}(\theta, X, Z)]$$

M-Step: Find the parameter that maximizes this quantity

$$\theta_{t+1} = \underset{\theta}{\operatorname{argmax}}\, Q(Q|\theta_t)$$

EM – In our Example

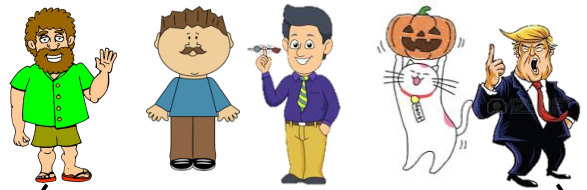
Initialize θ_0

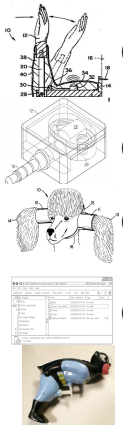
For t = 0,1,2,…

E-Step: Calculate the expected labels (e.g., bogus or not-bogus) given θ_t

M-Step: Given the estimated label, optimize θ and set it to θ_{t+1}

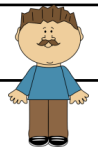
EM Algorithm - Example





$$\begin{array}{c} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{array} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

True \ Guess	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1



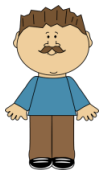
True \ Guess	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

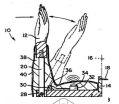
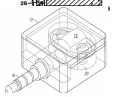
True \ Guess	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

True \ Guess	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

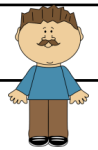
EM Algorithm - Example


$$\begin{array}{c} \\ o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{array} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{matrix} \text{Bogus} & \text{Not Bogus} \\ \begin{pmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \\ 0.4 & 0.6 \\ 0.2 & 0.8 \\ 0.6 & 0.4 \end{pmatrix} \end{matrix}$$

True \ Guess	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1



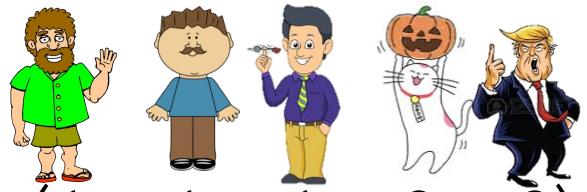
True \ Guess	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

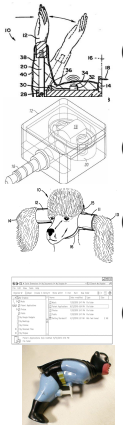
True \ Guess	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

True \ Guess	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

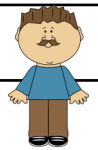
EM Algorithm - Example





$$\begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{matrix} \text{Bogus} & \text{Not Bogus} \\ \begin{pmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \\ 0.4 & 0.6 \\ 0.2 & 0.8 \\ 0.6 & 0.4 \end{pmatrix} \end{matrix} \quad \begin{matrix} \text{“Correct” Labels} \\ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \end{matrix}$$

True \ Guess	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1



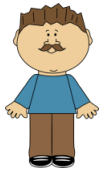
True \ Guess	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

True \ Guess	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

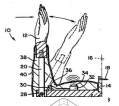
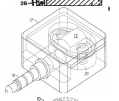
True \ Guess	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

EM Algorithm - Example


"Correct" Labels


$$o_1 \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
o_2 o_3 o_4 o_5

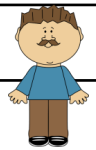
Guess

True	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

Guess

True	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?



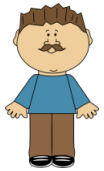
Guess

True	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

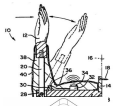
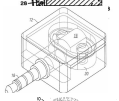
Guess

True	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

EM Algorithm - Example


"Correct" Labels

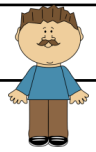




$$\begin{array}{c} \\ o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{array} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

True \ Guess	Bogus	!Bogus
Bogus	1	0.25
!Bogus	0	0.75

True \ Guess	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

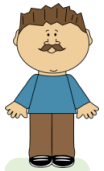
	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?



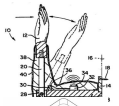
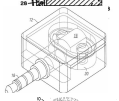
True \ Guess	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

True \ Guess	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

EM Algorithm - Example


"Correct" Labels


$$o_1 \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
o_2 o_3 o_4 o_5

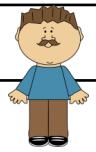
Guess

True	Bogus	!Bogus
Bogus	1	0.25
!Bogus	0	0.75

Guess

True	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?



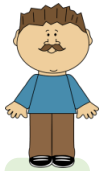
Guess

True	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

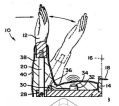
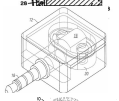
Guess

True	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

EM Algorithm - Example


"Correct" Labels


$$o_1 \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
o_2 o_3 o_4 o_5

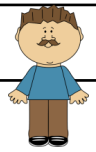
Guess

True	Bogus	!Bogus
Bogus	1	0.25
!Bogus	0	0.75

Guess

True	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?



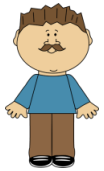
Guess

True	Bogus	!Bogus
Bogus	0.66	0
!Bogus	0.33	1

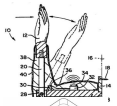
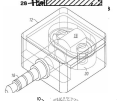
Guess

True	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

EM Algorithm - Example


"Correct" Labels

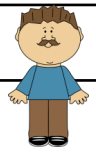




$$\begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

True \ Guess	Bogus	!Bogus
Bogus	1	0.25
!Bogus	0	.75

True \ Guess	Bogus	!Bogus
Bogus	.66	0
!Bogus	.33	1

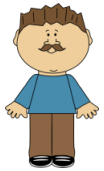
	Bogus	!Bogus
Bogus	0	0.66
!Bogus	1	0.33



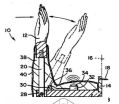
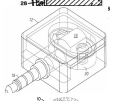
True \ Guess	Bogus	!Bogus
Bogus	0.66	0
!Bogus	0.33	1

True \ Guess	Bogus	!Bogus
Bogus	.5	0.33
!Bogus	.5	0.66

EM Algorithm - Example


"Correct" Labels


$$o_1 \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{pmatrix}$$
o_2
o_3
o_4
o_5

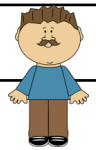
True \ Guess

	Bogus	!Bogus
Bogus	1	0.25
!Bogus	0	.75

True \ Guess

	Bogus	!Bogus
Bogus	.66	0
!Bogus	.33	1

	Bogus	!Bogus
Bogus	0	0.66
!Bogus	1	0.33



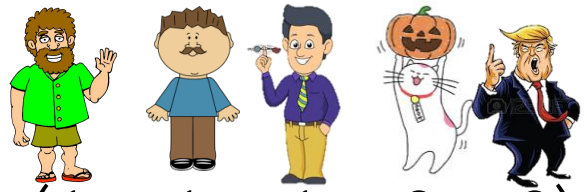
True \ Guess

	Bogus	!Bogus
Bogus	0.66	0
!Bogus	0.33	1

True \ Guess

	Bogus	!Bogus
Bogus	.5	0.33
!Bogus	.5	0.66

EM Algorithm - Example



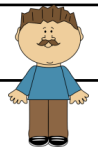
o_1	1	1	1	0	0
o_2	0	0	0	1	1
o_3	0	1	1	0	0
o_4	0	0	0	0	1
o_5	0	1	1	1	0

Bogus	Not Bogus
$1 + .66 + .66 + .33 + .66$	$0 + .33 + .33 + .66 + .33$
$0.25 + 0 + 0 + .5 + 0$	$.75 + 1 + 1 + 0.5 + 1$
$0.25 + .66 + .66 + 0.33 + .66$	$.75 + .33 + .33 + 0.66 + .33$
$0.25 + 0 + 0 + .33 + 0$	$.75 + 1 + 1 + .66 + 1$
$.25 + .66 + .66 + .5 + .66$	$.75 + .33 + .33 + .5 + .33$

True \ Guess	Bogus	!Bogus
Bogus	1	0.25
!Bogus	0	.75

True \ Guess	Bogus	!Bogus
Bogus	.66	0
!Bogus	.33	1

	Bogus	!Bogus
Bogus	0	0.66
!Bogus	1	0.33



True \ Guess	Bogus	!Bogus
Bogus	0.66	0
!Bogus	0.33	1

True \ Guess	Bogus	!Bogus
Bogus	.5	0.33
!Bogus	.5	0.66

EM Algorithm - Example

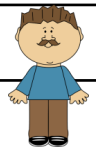
o_1	1	1	1	0	0
o_2	0	0	0	1	1
o_3	0	1	1	0	0
o_4	0	0	0	0	1
o_5	0	1	1	1	0

Bogus	Not Bogus
$1 + .66 + .66 + .33 + .66$	$0 + .33 + .33 + .66 + .33$
$0.25 + 0 + 0 + .5 + 0$	$.75 + 1 + 1 + 0.5 + 1$
$0.25 + .66 + .66 + 0.33 + .66$	$.75 + .33 + .33 + 0.66 + .33$
$0.25 + 0 + 0 + .33 + 0$	$.75 + 1 + 1 + .66 + 1$
$.25 + .66 + .66 + .5 + .66$	$.75 + .33 + .33 + .5 + .33$

True \ Guess	Bogus	!Bogus
Bogus	1	0.25
!Bogus	0	.75

True \ Guess	Bogus	!Bogus
Bogus	.66	0
!Bogus	.33	1

	Bogus	!Bogus
Bogus	0	0.66
!Bogus	1	0.33



True \ Guess	Bogus	!Bogus
Bogus	0.66	0
!Bogus	0.33	1

True \ Guess	Bogus	!Bogus
Bogus	.5	0.33
!Bogus	.5	0.66

EM Algorithm - Example

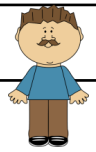
o_1	1	1	1	0	0
o_2	0	0	0	1	1
o_3	0	1	1	0	0
o_4	0	0	0	0	1
o_5	0	1	1	1	0

Bogus	Not Bogus
1 + .66 + .66 + .33 + .66	0 + .33 + .33 + .66 + .33
0.25 + 0 + 0 + .5 + 0	.75 + 1 + 1 + 0.5 + 1
0.25 + .66 + .66 + 0.33 + .66	.75 + .33 + .33 + 0.66 + .33
0.25 + 0 + 0 + .33 + 0	.75 + 1 + 1 + .66 + 1
.25 + .66 + .66 + .5 + .66	.75 + .33 + .33 + .5 + .33

True \ Guess	Bogus	!Bogus
Bogus	1	0.25
!Bogus	0	.75

True \ Guess	Bogus	!Bogus
Bogus	.66	0
!Bogus	.33	1

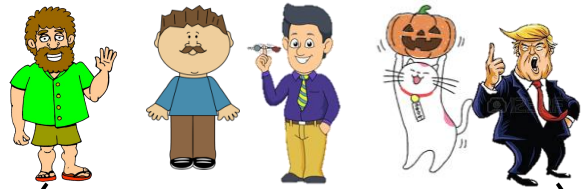
	Bogus	!Bogus
Bogus	0	0.66
!Bogus	1	0.33



True \ Guess	Bogus	!Bogus
Bogus	0.66	0
!Bogus	0.33	1

True \ Guess	Bogus	!Bogus
Bogus	.5	0.33
!Bogus	.5	0.66

EM Algorithm - Example

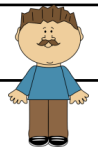


						Bogus	Not Bogus	"Correct" Labels
o_1	1	1	1	0	0	0.66	.33	1
o_2	0	0	0	1	1	0.15	.85	0
o_3	0	1	1	0	0	0.52	0.48	1
o_4	0	0	0	0	1	.12	0.88	0
o_5	0	1	1	1	0	0.55	0.45	1

True \ Guess	Bogus	!Bogus
Bogus	1	0.25
!Bogus	0	.75

True \ Guess	Bogus	!Bogus
Bogus	.66	0
!Bogus	.33	1

	Bogus	!Bogus
Bogus	0	0.66
!Bogus	1	0.33



True \ Guess	Bogus	!Bogus
Bogus	0.66	0
!Bogus	0.33	1

True \ Guess	Bogus	!Bogus
Bogus	.5	0.33
!Bogus	.5	0.66

Dawid and Skene EM Algorithm [1]

Input: Labels $l[k][n]$ from worker (k) to object o_n,

Output: Confusion matrix $\pi_{ij}^{(k)}$ for each worker (k), Correct labels $T(o_n)$ for each object o_n, Class priors $Pr\{C\}$ for each class C

1 Initialize error rates $\pi_{ij}^{(k)}$ for each worker (k) (e.g., assume each worker is perfect);

2 Initialize correct label for each object $T(o_n)$ (e.g., using majority vote);

3 **while** *not converged* **do**

4 Estimate the correct label $T(o_n)$ for each object, using the labels $l[\cdot][n]$ assigned to o_n by workers, weighting the votes using the error rates $\pi_{ij}^{(k)}$;

5 Estimate the error rates $\pi_{ij}^{(k)}$, for each worker (k), using the correct labels $T(o_n)$ and the assigned labels $l[k][n]$;

6 Estimate the class priors $Pr\{C\}$, for each class C;

7 **end**

8 **return** *Estimated error rates* $\pi_{ij}^{(k)}$, *Estimated correct labels* $T(o_n)$, *Estimated class priors* $Pr\{C\}$

[1] Panos Ipeirotis, Foster Provost, Jing Wang: **Quality management on Amazon Mechanical Turk.** Proceedings of the ACM SIGKDD Workshop on Human Computation, 2010

[2] Dawid, A. P., and Skene, A. M. **Maximum likelihood estimation of observer error-rates using the EM algorithm**. Applied Statistics 28, 1 (Sept. 1979), 20–28.

Confusion Matrices in the 2nd iteration

True	Bogus	!Bogus
Bogus	1	0.5
!Bogus	0	.5

True	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

True	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

True	Bogus	!Bogus
Bogus	.5	0.66
!Bogus	.5	0.33

	Bogus	!Bogus
Bogus	0	1
!Bogus	1	0

Which worker is the worst?

EM-Algorithm: Many other applications

Initialize $\theta \in \Theta$

For t = 0,1,2,…

E-Step: Calculate the expected value of the log likelihood function, with respect to the conditional distribution of Z given X under the current estimate of the parameters θ_t:

$$Q(Q|\theta_t) = E_{Z|X,\theta_t}[\log \mathcal{L}(\theta, X, Z)]$$

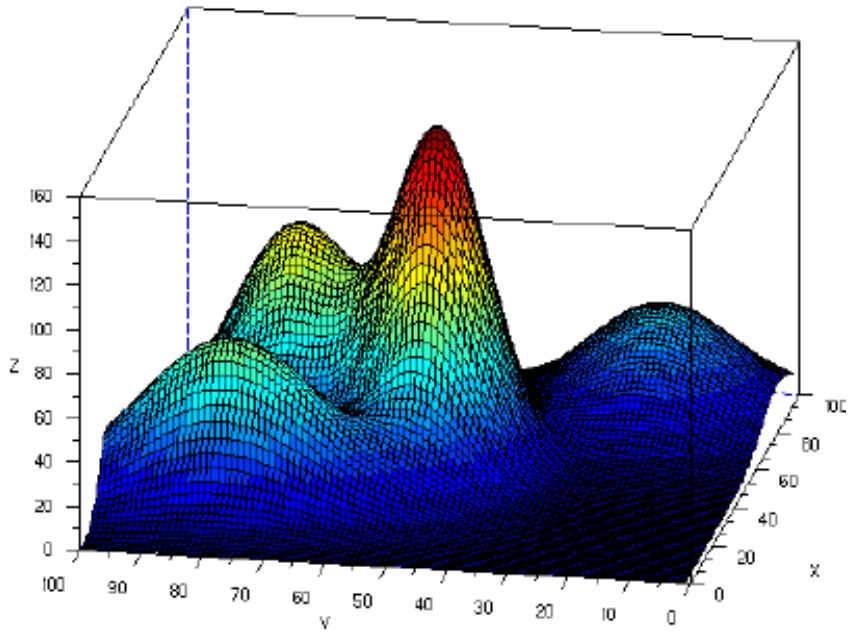
M-Step: Find the parameter that maximizes this quantity

$$\theta_{t+1} = \underset{\theta}{\operatorname{argmax}}\, Q(Q|\theta_t)$$

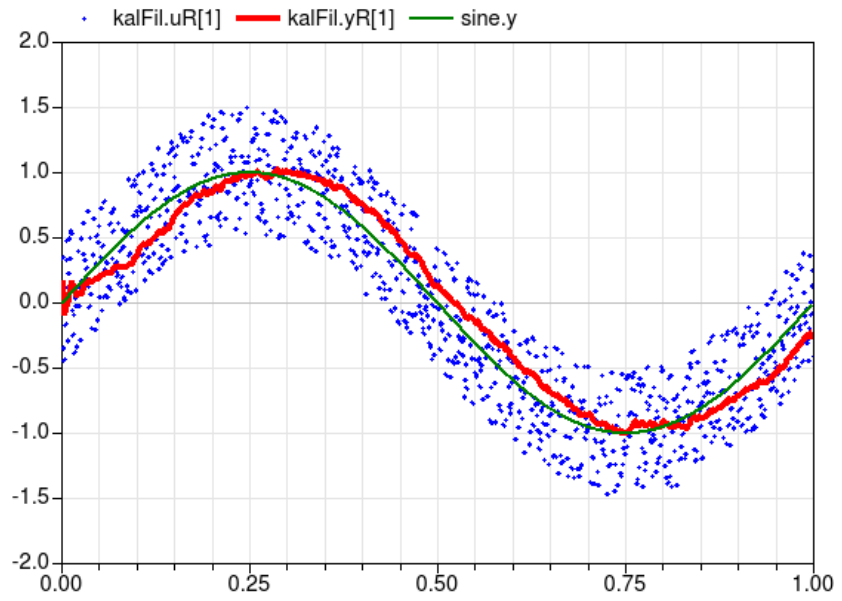
EM-Algorithm:
Many other applications

Initialize $\theta \in \Theta$

Gaussian Mixture Models (GMM)



Kalman filter



M step: Find the parameter that maximizes this quantity

$$\theta_{t+1} = \underset{\theta}{\operatorname{argmax}}\, Q\left(Q\middle|\theta_t\right)$$

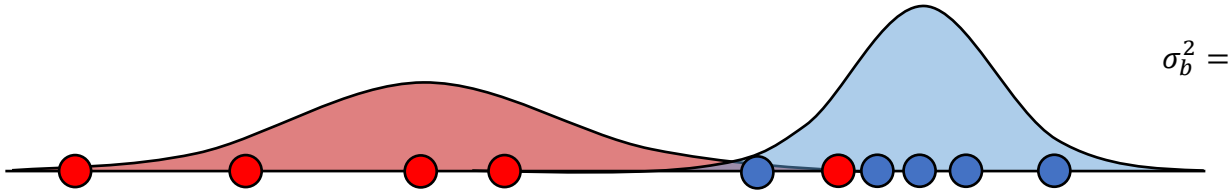
EM-Algorithm: Gaussian Mixture Models

Unknown distributions parameters, known data point labels

- K=2 Gaussians with unknown **μ**, **σ**
- Assume you know if data point comes from the red or blue distribution.
- Estimating the distribution parameters is trivial

$$\mu_b = \frac{x_1 + x_2 + \cdots + x_n}{n_b}$$

$$\sigma_b^2 = \frac{(x_1 - \mu_b)^2 + (x_2 - \mu_b)^2 + \cdots + n - \mu_b)^2}{n_b}$$



EM-Algorithm: Gaussian Mixture Models

Unknown distributions parameters, known data point labels

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- Assume you know if data point comes from the red or blue distribution.
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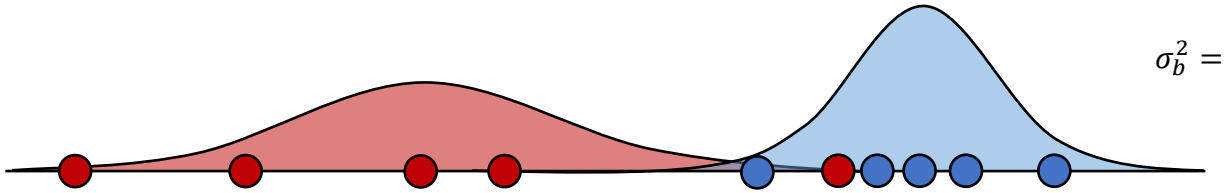
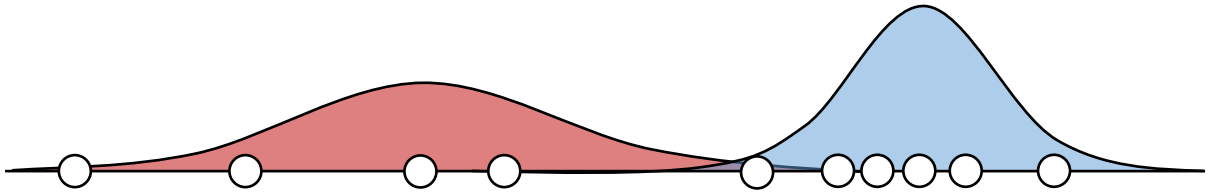
$$\mu_b = \frac{x_1 + x_2 + \cdots + x_n}{n_b}$$

$$\sigma_b^2 = \frac{(x_1 - \mu_b)^2 + (x_2 - \mu_b)^2 + \cdots + (x_n - \mu_b)^2}{n_b}$$



Known distributions parameters, unknown data point labels

- We can guess weather the point is more likely from the blue or red distribution



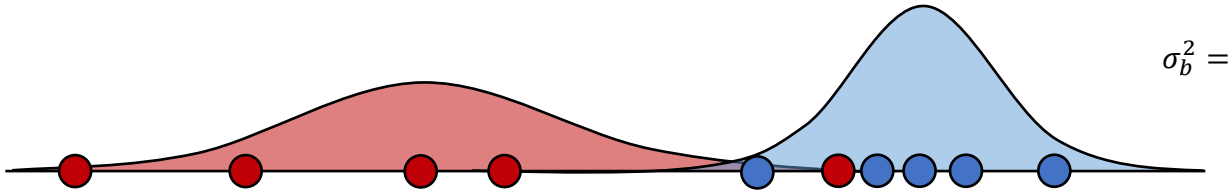
EM-Algorithm: Gaussian Mixture Models

Unknown distributions parameters, known data point labels

- K=2 Gaussians with unknown **μ**, **σ**
- Assume you know if data point comes from the red or blue distribution.
- Estimating the distribution parameters is trivial

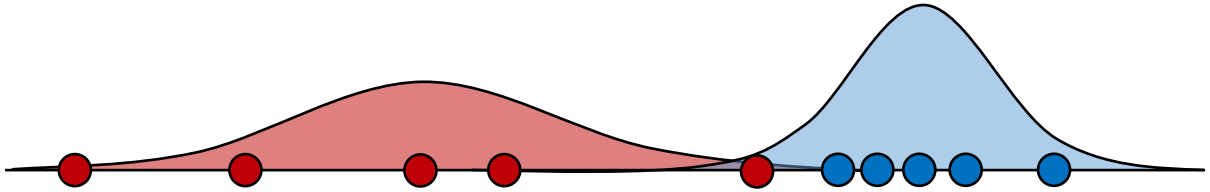
$$\mu_b = \frac{x_1 + x_2 + \cdots + x_n}{n_b}$$

$$\sigma_b^2 = \frac{(x_1 - \mu_b)^2 + (x_2 - \mu_b)^2 + \cdots + n - \mu_b)^2}{n_b}$$



Known distributions parameters, unknown data point labels

- We can guess weather the point is more likely from the blue or red distribution



$$P(b|x_1) = \frac{P(x_1|b)P(b)}{P(x_1|b)P(b) + P(x_1|r)P(r)}$$

$$P(x_1|b) = \frac{1}{\sqrt{2\pi\sigma_b^2}} \exp\left(-\frac{(x-\mu_b)^2}{2\sigma_b^2}\right)$$

EM-Algorithm: Gaussian Mixture Models

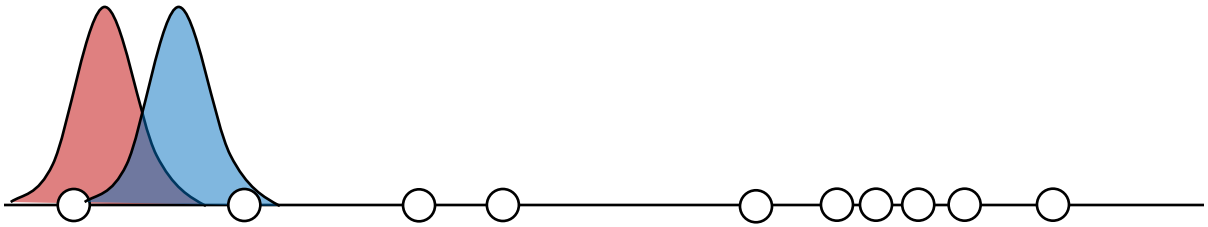
Chicken and egg problem

- Need(μ_r, σ_r^2) and (μ_b, σ_b^2) to guess source of points
- Need to know the source to estimate (μ_r, σ_r^2) and (μ_b, σ_b^2)

EM algorithm

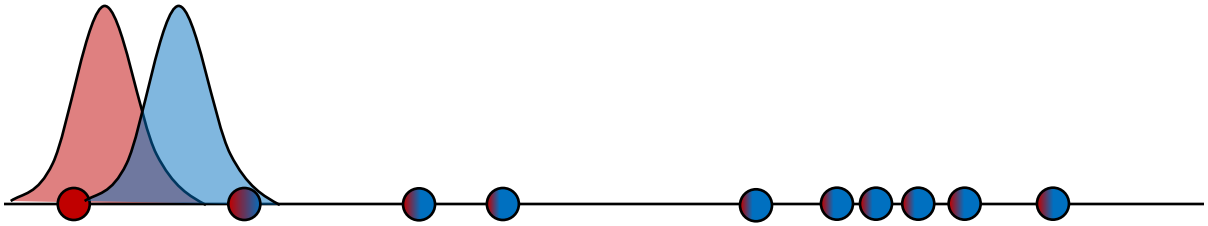
- Start with two randomly placed Gaussians (μ_r, σ_r^2) and (μ_b, σ_b^2)
- E-step: For each point: $P(b|x_1)$ = does it look it came from blue (red)
- M-Step: adjust (μ_r, σ_r^2) and (μ_b, σ_b^2) to fit points assigned to them
- Iterate until convergence

EM-Algorithm: Gaussian Mixture Models



- Start with two randomly placed Gaussians (μ_r, σ_r^2) and (μ_b, σ_b^2)

EM-Algorithm: Gaussian Mixture Models



- Start with two randomly placed Gaussians (μ_r, σ_r^2) and (μ_b, σ_b^2)
- E-step: For each point: $P(b|x_1)$ = does it look it came from blue (red)

E-Step:

$$P(x_1|b) = \frac{1}{\sqrt{2\pi\sigma_b^2}} \exp\left(-\frac{(x-\mu_b)^2}{2\sigma_b^2}\right)$$

Note: we could estimate priors P(b) and P(r), but often left at equal chance

$$b_i = P(b|x_i) = \frac{P(x_i|b)P(b)}{P(x_i|b)P(b) + P(x_i|r)P(r)}$$

$$r_i = 1 - b_i$$

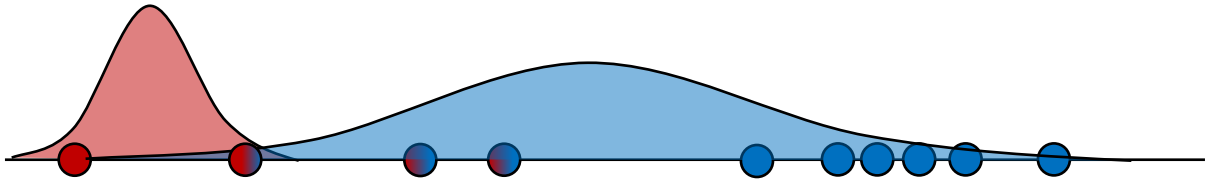
- M-Step: adjust (μ_r, σ_r^2) and (μ_b, σ_b^2) to fit points assigned to them

M-Step:

$$\mu_b = \frac{b_1 x_1 + b_2 x_2 + \cdots + b_n x_n}{b_1 + b_2 + \cdots + b_n}$$

$$\sigma_b^2 = \frac{b_1(x_1-\mu_b)^2 + b_2(x_2-\mu_b)^2 + \cdots + b_n(n-\mu_b)^2}{n_b}$$

EM-Algorithm: Gaussian Mixture Models



- Start with two randomly placed Gaussians (μ_r, σ_r^2) and (μ_b, σ_b^2)
- E-step: For each point: $P(b|x_1)$ = does it look it came from blue (red)

E-Step:

$$P(x_1|b) = \frac{1}{\sqrt{2\pi\sigma_b^2}} \exp\left(-\frac{(x-\mu_b)^2}{2\sigma_b^2}\right)$$

Note: we could estimate priors P(b) and P(r), but often left at equal chance

$$b_i = P(b|x_i) = \frac{P(x_i|b)P(b)}{P(x_i|b)P(b) + P(x_i|r)P(r)}$$

$$r_i = 1 - b_i$$

- M-Step: adjust (μ_r, σ_r^2) and (μ_b, σ_b^2) to fit points assigned to them

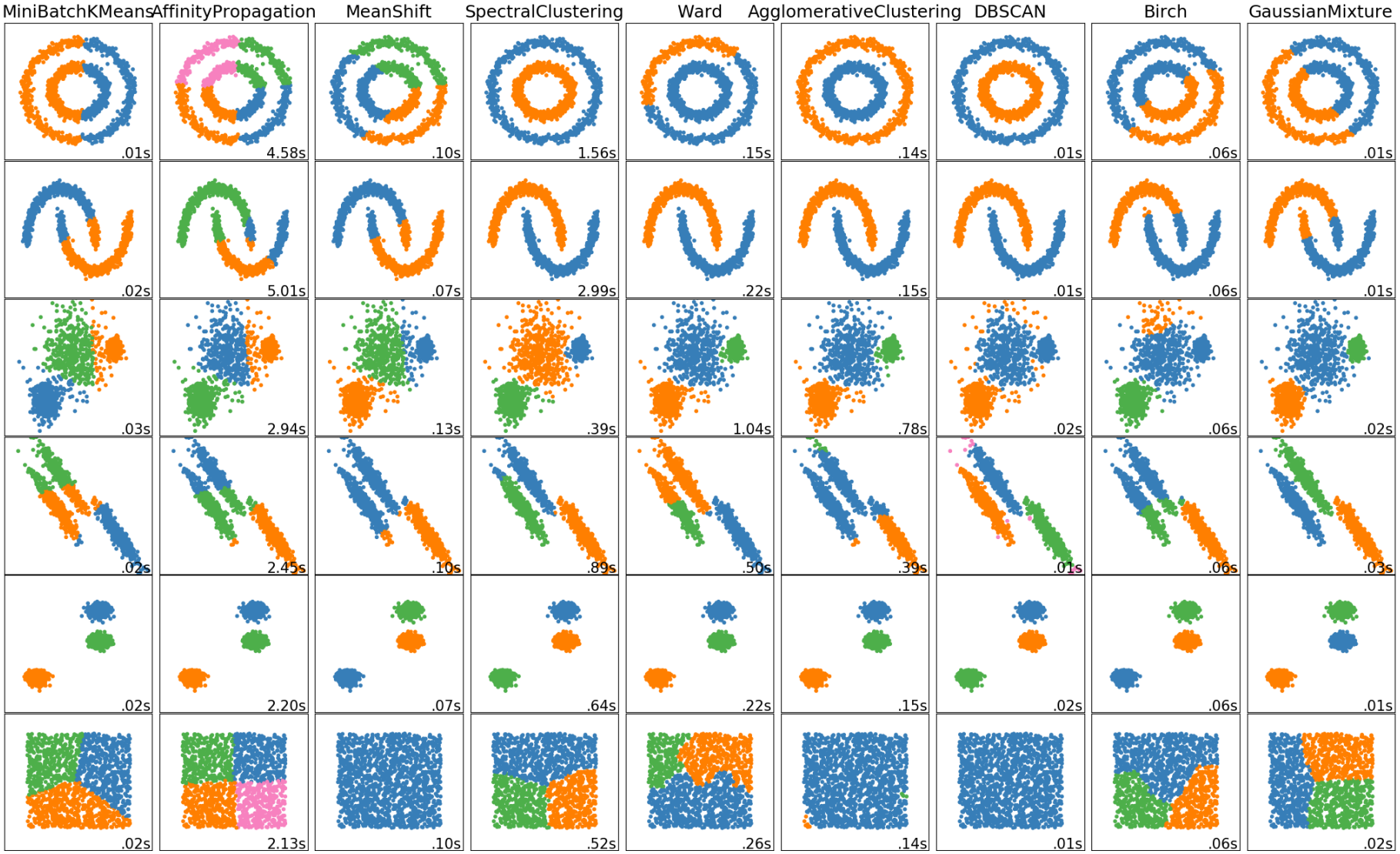
M-Step:

$$\mu_b = \frac{b_1 x_1 + b_2 x_2 + \cdots + b_n x_n}{b_1 + b_2 + \cdots + b_n}$$

$$\sigma_b^2 = \frac{b_1(x_1-\mu_b)^2 + b_2(x_2-\mu_b)^2 + \cdots + b_n(n-\mu_b)^2}{n_b}$$

In what way is the algorithm similar to k-means and in what ways is it different?

CLUSTERING


MiniBatchKMeans
AffinityPropagation
MeanShift
SpectralClustering
Ward
AgglomerativeClustering
DBSCAN
Birch
GaussianMixture
.01s
4.58s
.10s
1.56s
.15s
.14s
.01s
.06s
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.02s
5.01s
.07s
2.99s
.22s
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2.94s
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.39s
1.04s
.78s
.02s
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.02s
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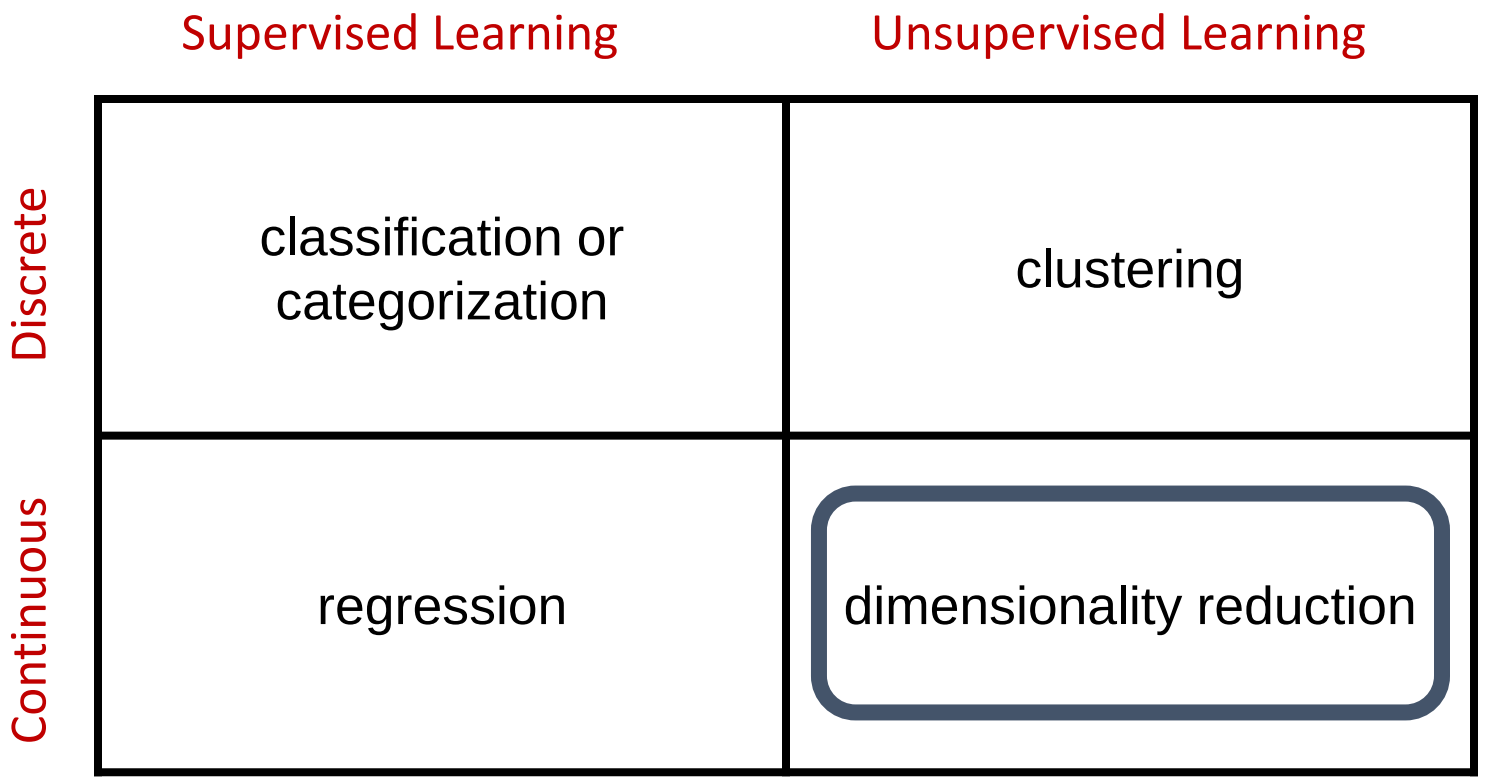
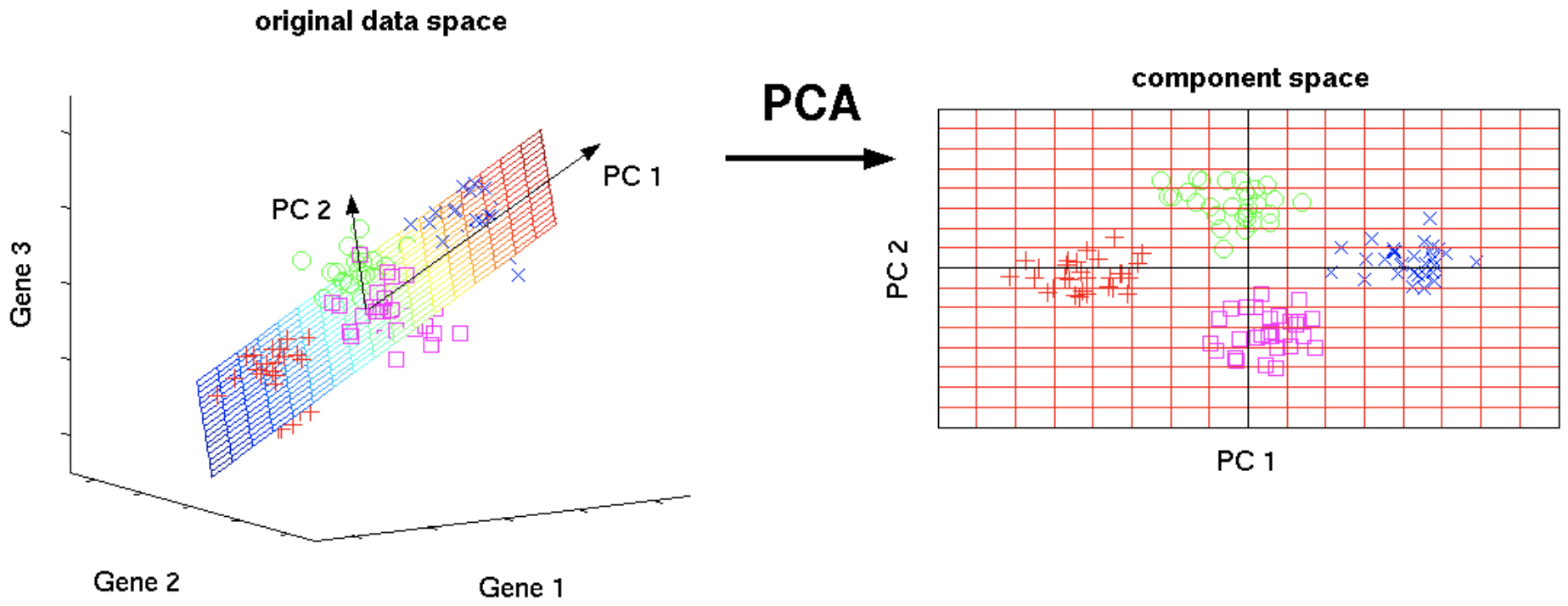
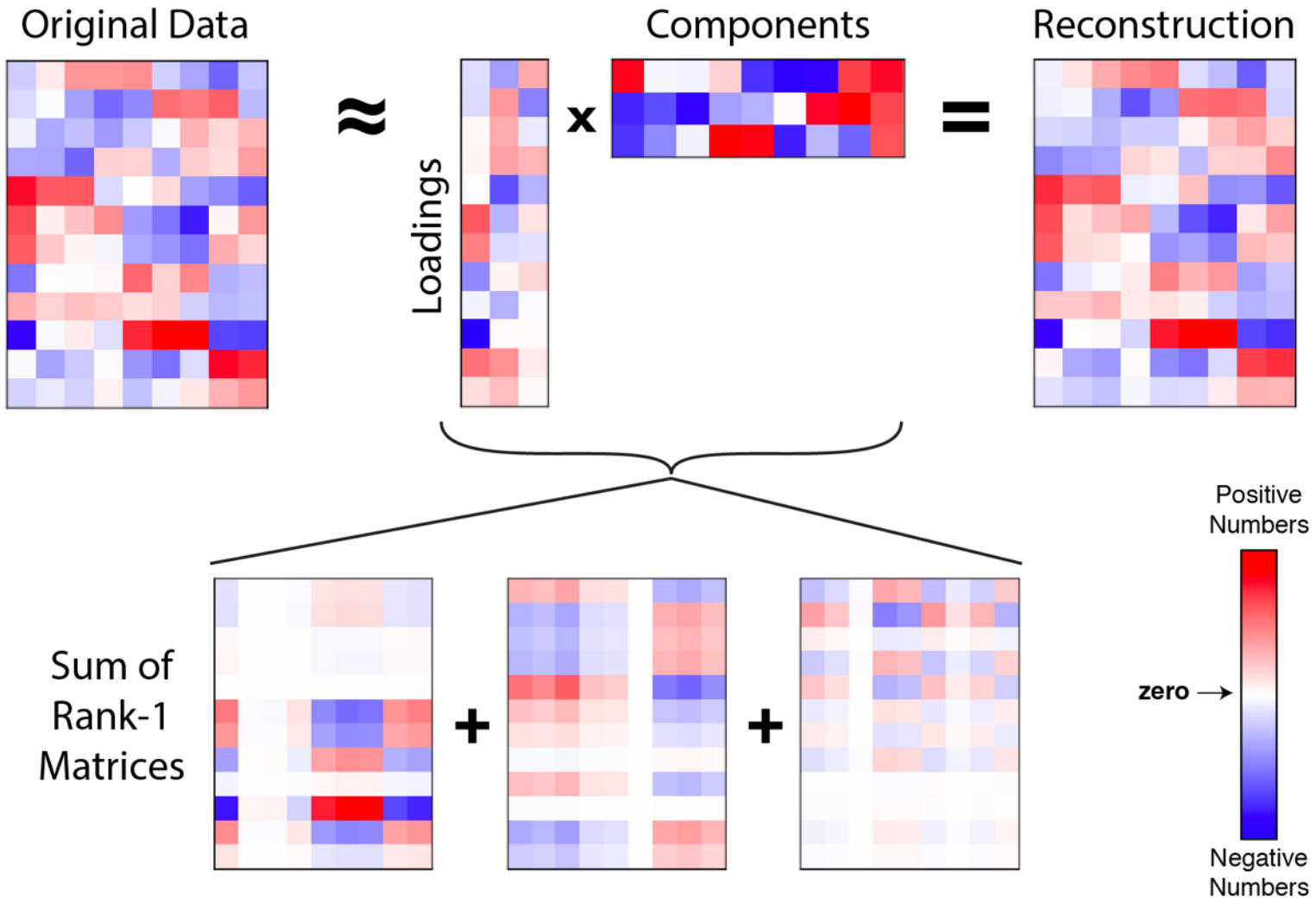
Machine Learning Problems

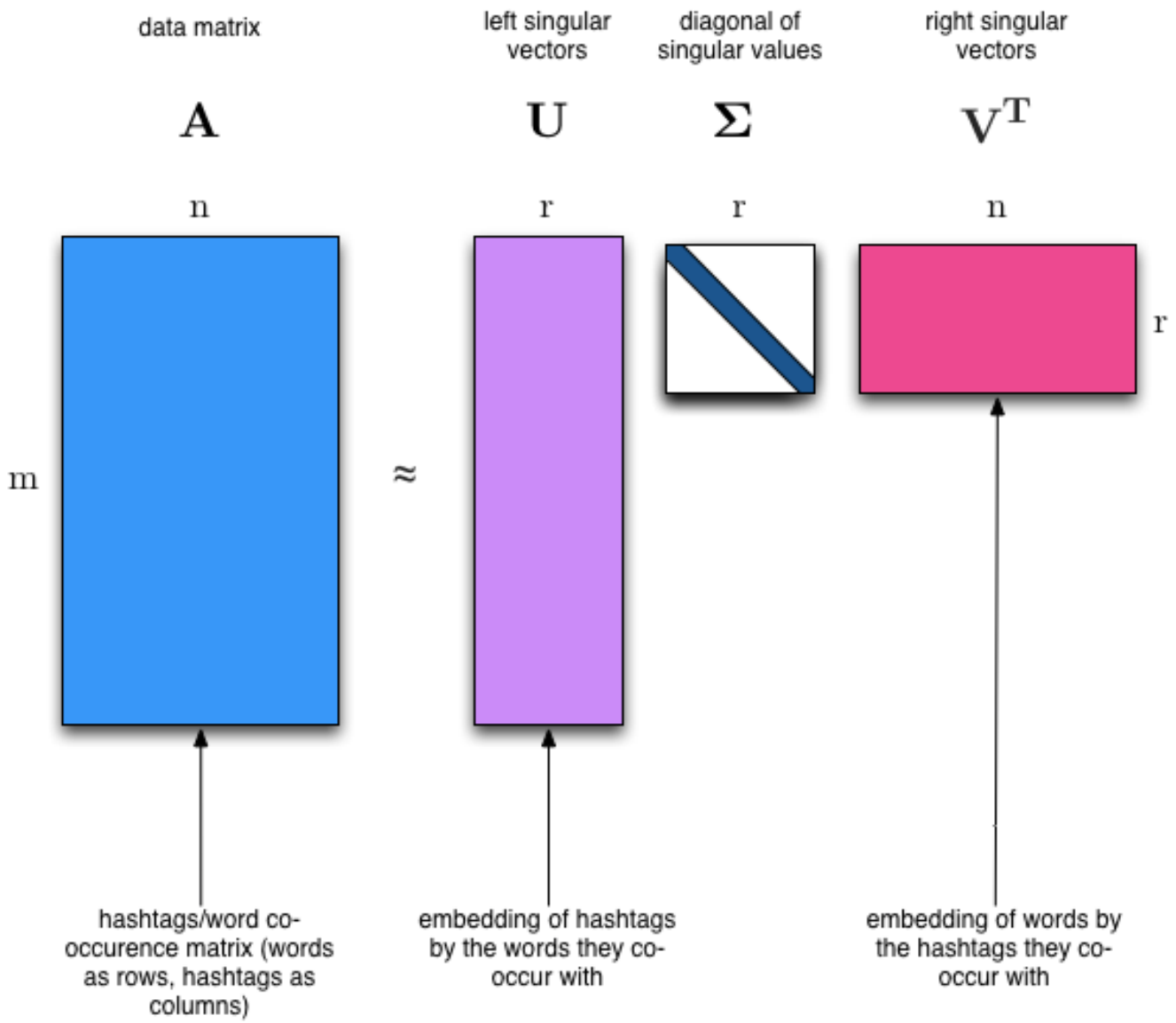


PCA


original data space
PC 1
PC 2
Gene 3
Gene 2
Gene 1
PCA
component space
PC 2
PC 1

PCA Intuition




data matrix
left singular vectors
diagonal of singular values
right singular vectors
A
U
Σ
V^T
n
r
r
n
m
≈
r
hashtags/word co-occurence matrix (words as rows, hashtags as columns)
embedding of hashtags by the words they co-occur with
embedding of words by the hashtags they co-occur with

PRINCIPAL COMPONENT ANALYSIS (PCA)