6.S079 MACHINE LEARNING 1

MARCH 5, 2024
MIKE CAFARELLA

THANKS TO TIM KRASKA FOR SLIDES

MACHINE LEARNING PROBLEMS

(Boosted-) Decision Trees

K-Means
Agglomerative clustering
DBScan

	Supervised Learning	Unsupervised Learning
Discrete	classification or categorization	clustering
Continuous	regression	dimensionality reduction

(Boosted-) Decision Trees

PCA

CLUSTERING STRATEGIES

K-means

- Iteratively re-assign points to the nearest cluster center

Agglomerative clustering

- Start with each point as its own cluster and iteratively merge the closest clusters

DBSCAN (Density-based spatial clustering of applications with noise)

EM Algorithm and Mixture Gaussian clustering

K-MEANS

Lloyd's Algorithm is the most common, naïve approach

1. Choose k cluster centers. Place them randomly
2. Repeatedly:
    1. Find the data points closest to each center, assign them to that center's cluster
    2. Compute the centroid of each cluster
    3. Move each cluster's center to its centroid
3. Terminate when points don't move much

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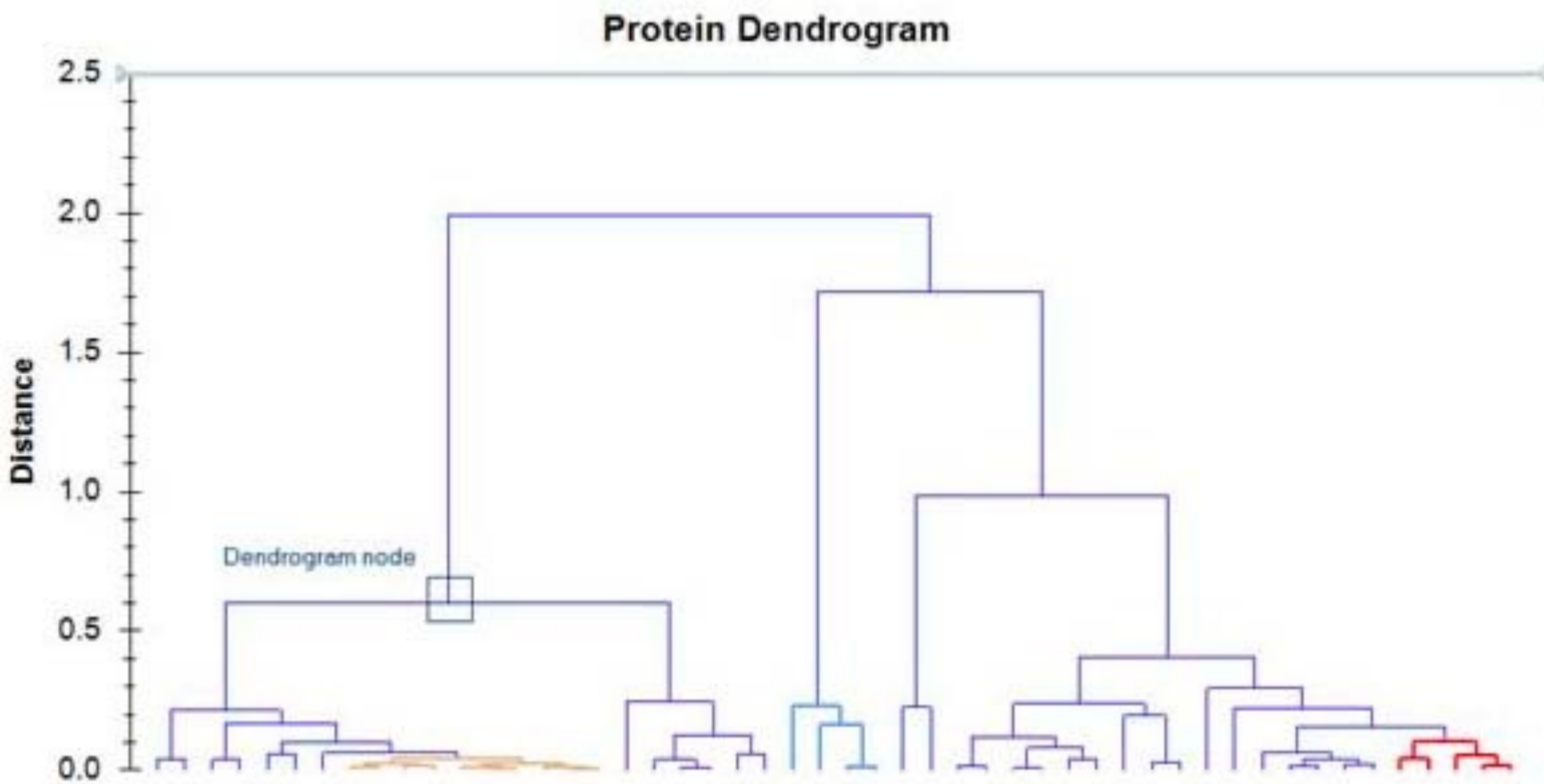
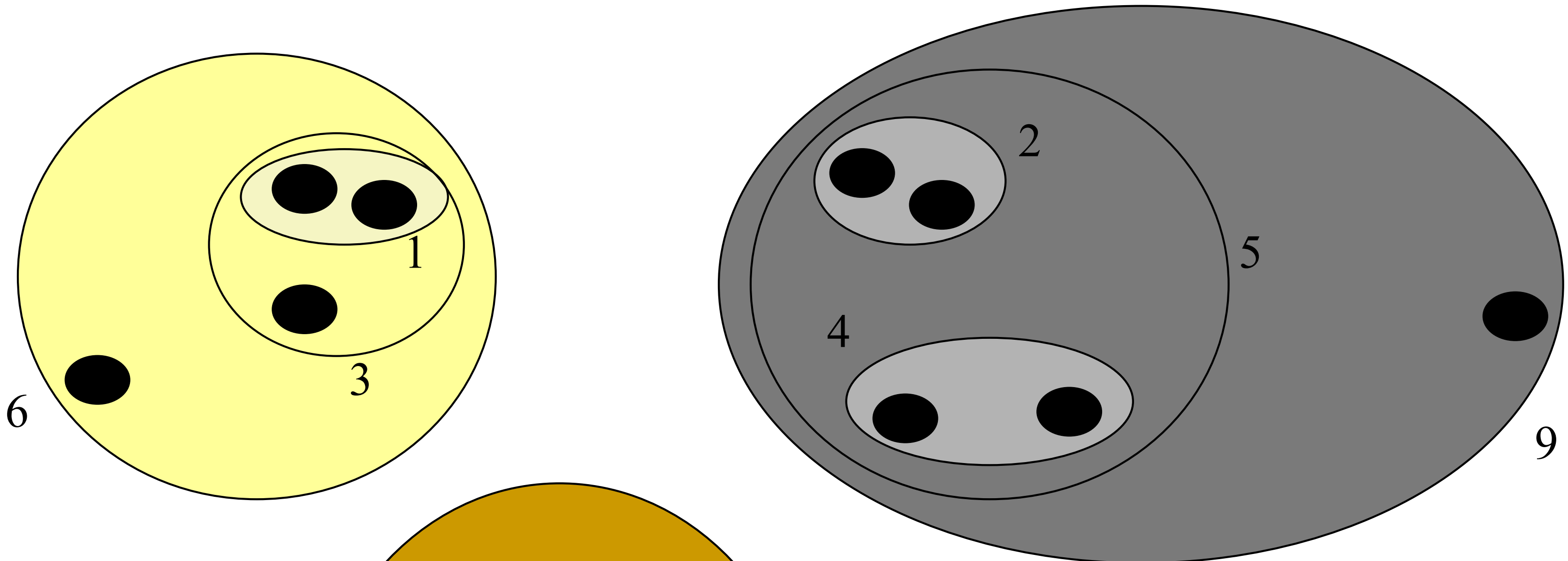
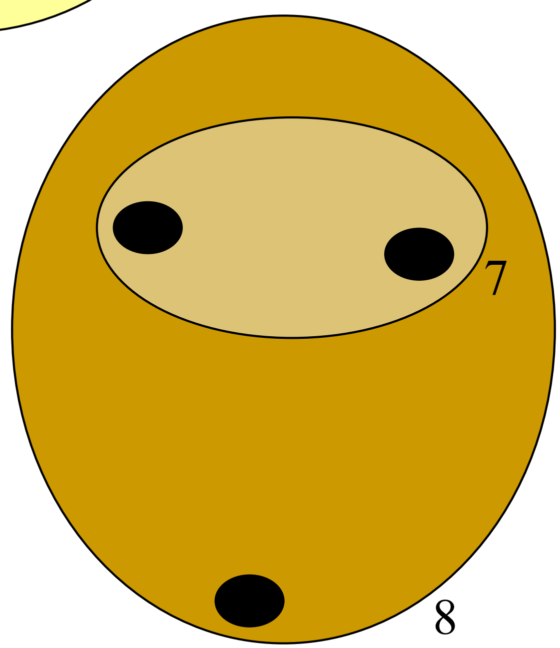
DENDROGRAM EXAMPLE



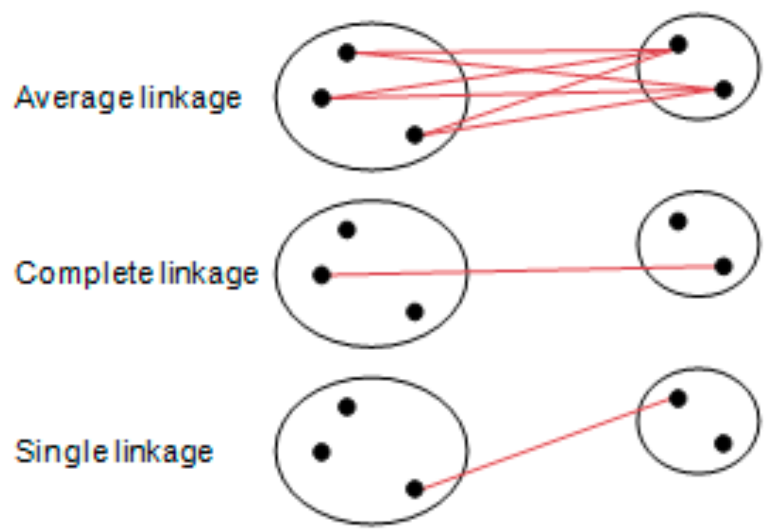
Group Agglomerative Clustering





Which linkage scheme potentially yields long, skinny clusters?

Which yields compact clusters?



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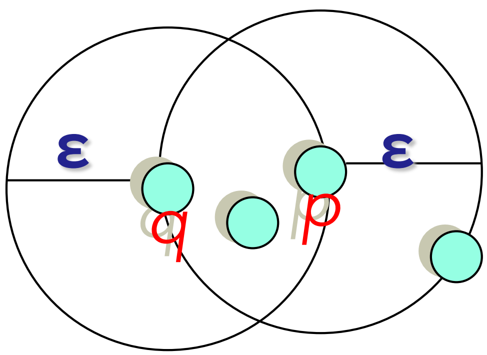
ε-NEIGHBORHOOD

ε-Neighborhood – Objects within a radius of *ε* from an object.

$$N_{\varepsilon}(p):\{q \mid d(p,q) \leq \varepsilon\}$$

"High density" - ε-Neighborhood of an object contains at least *MinPts* of objects.

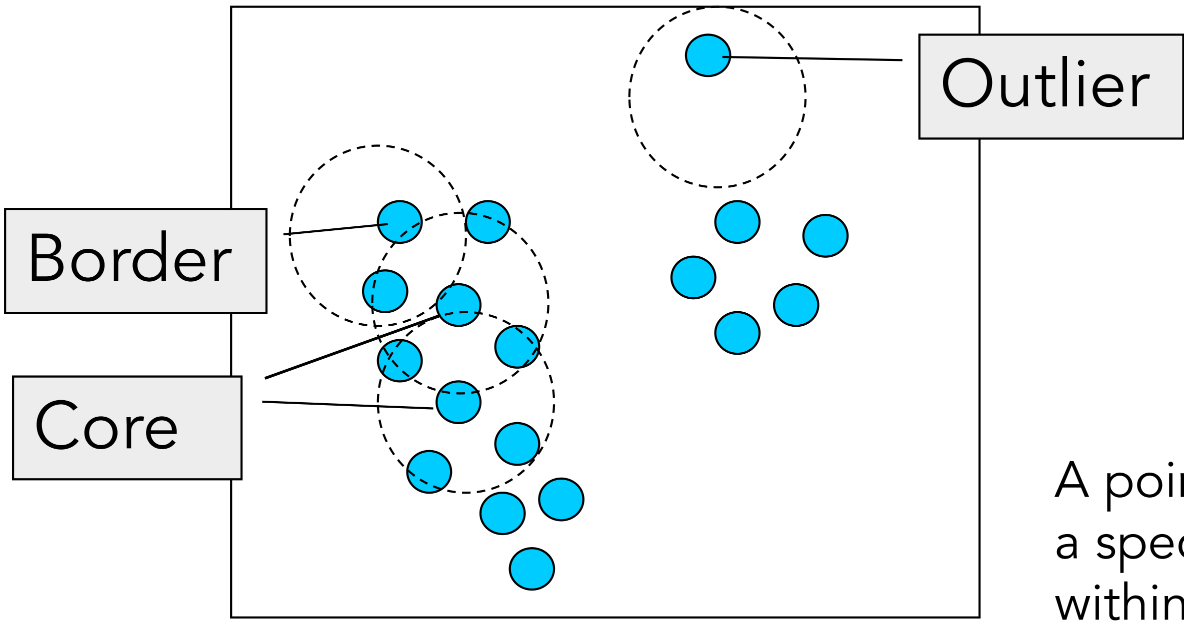


ε-Neighborhood of *p*
ε-Neighborhood of *q*

Density of p is "high" (MinPts = 4)

Density of q is "low" (MinPts = 4)

CORE, BORDER & OUTLIER (NOISE)



ε = 1unit, MinPts = 5

Given *ε* and *MinPts*, categorize the objects into three exclusive groups.

A point is a core point if it has more than a specified number of points (MinPts) within Epsilon. These are points that are at the interior of a cluster.

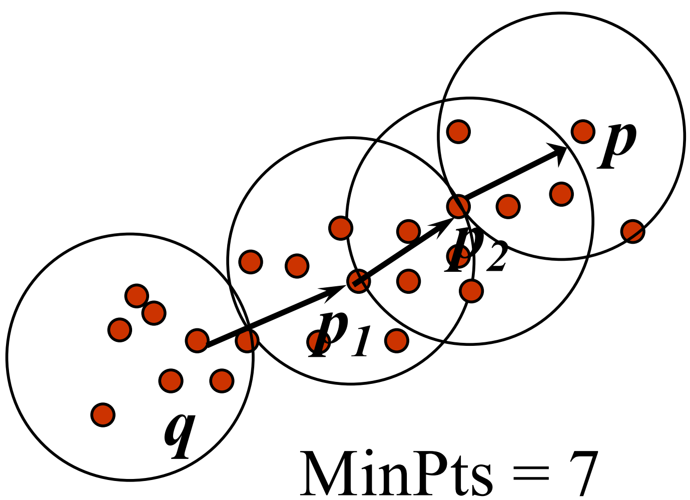
A border point has fewer than MinPts within Epsilon, but is in the neighborhood of a core point..

A noise point (outlier) is any point that is not a core point nor a border point.

DENSITY-REACHABILITY

Density-Reachable (directly and indirectly):

- A point p is directly density-reachable from p2;
- p2 is directly density-reachable from p1;
- p1 is directly density-reachable from q;
- p←p2←p1←q form a chain.



p is (indirectly) density-reachable from q

q is not density- reachable from p?

DBSCAN ALGORITHM

Input: The data set D

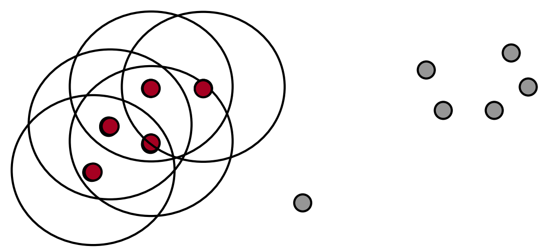
Parameter: ε, MinPts

```
For each object p in D
    if p is a core object and not processed then
        C = retrieve all objects density-reachable from p
        mark all objects in C as processed
        report C as a cluster
    else mark p as outlier
    end if
End For
```

DBSCAN ALGORITHM: EXAMPLE

Parameter

- ε = 2 cm
- *MinPts* = 3



for each $o \in D$ **do**
    if o is not yet classified **then**
        if o is a core-object **then**
            collect all objects density-reachable from o
            and assign them to a new cluster.
        else
            assign o to NOISE

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Parameter

- ε = 2 cm
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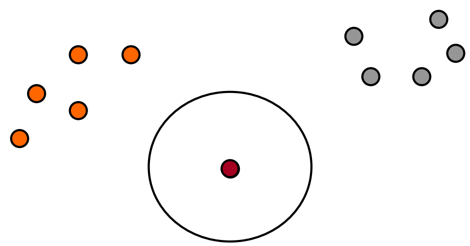

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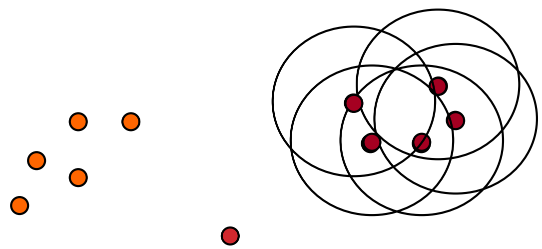

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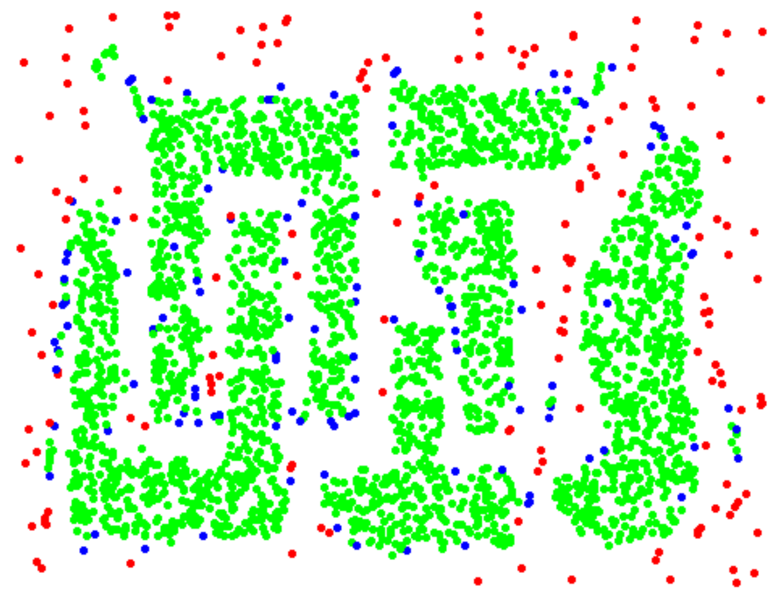
EXAMPLE

Original Points

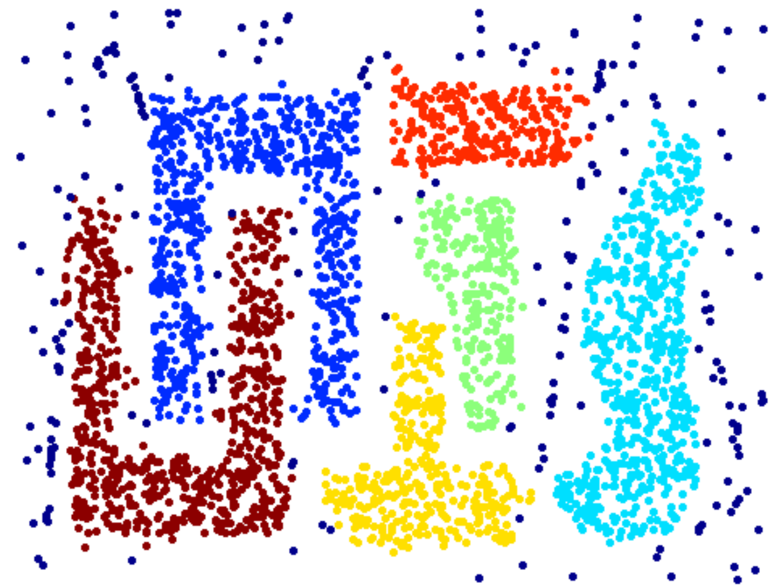


Point types: core, border and outliers

ε = 10, MinPts = 4

WHEN DBSCAN WORKS WELL

Original Points

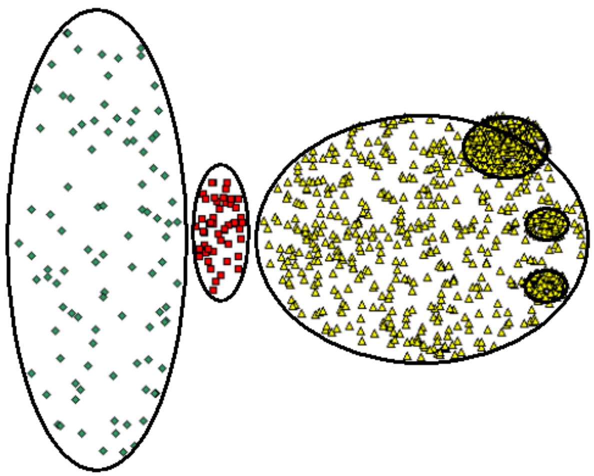


Clusters

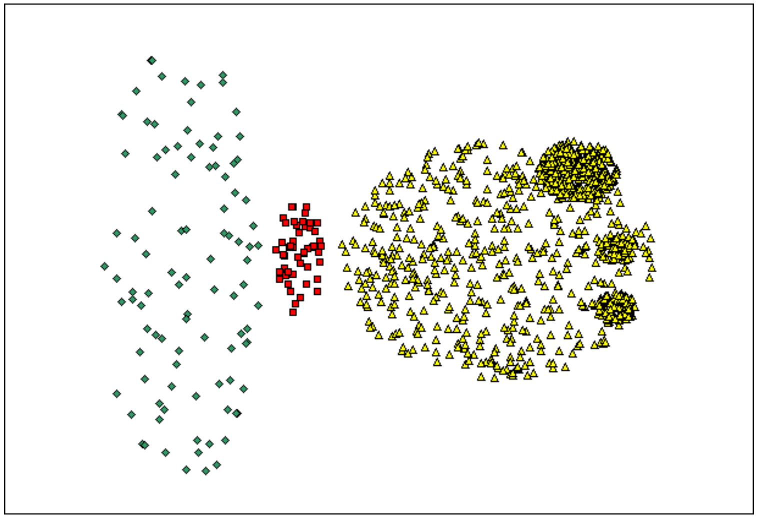
- Resistant to Noise
- Can handle clusters of different shapes and sizes
- You don’t need to specify the number of clusters in advance

CAN YOU CREATE AN EXAMPLE FOR WHICH DBSCAN WILL NOT WORK WELL?

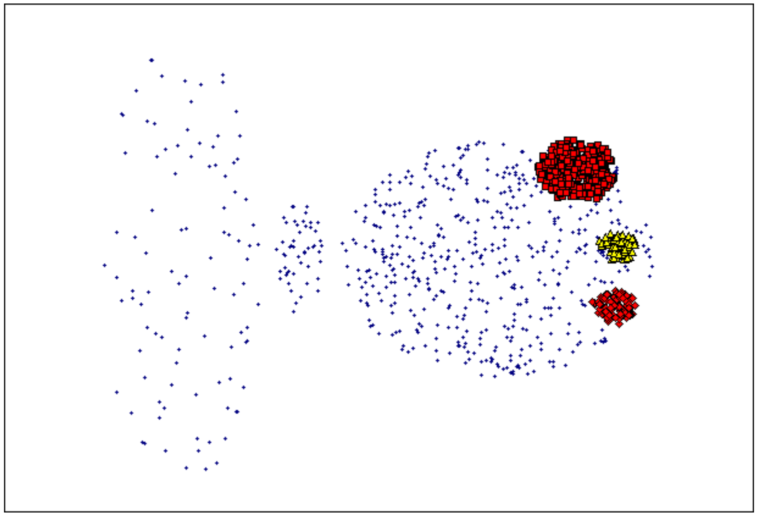
WHEN DBSCAN DOES NOT WORK WELL



Original Points



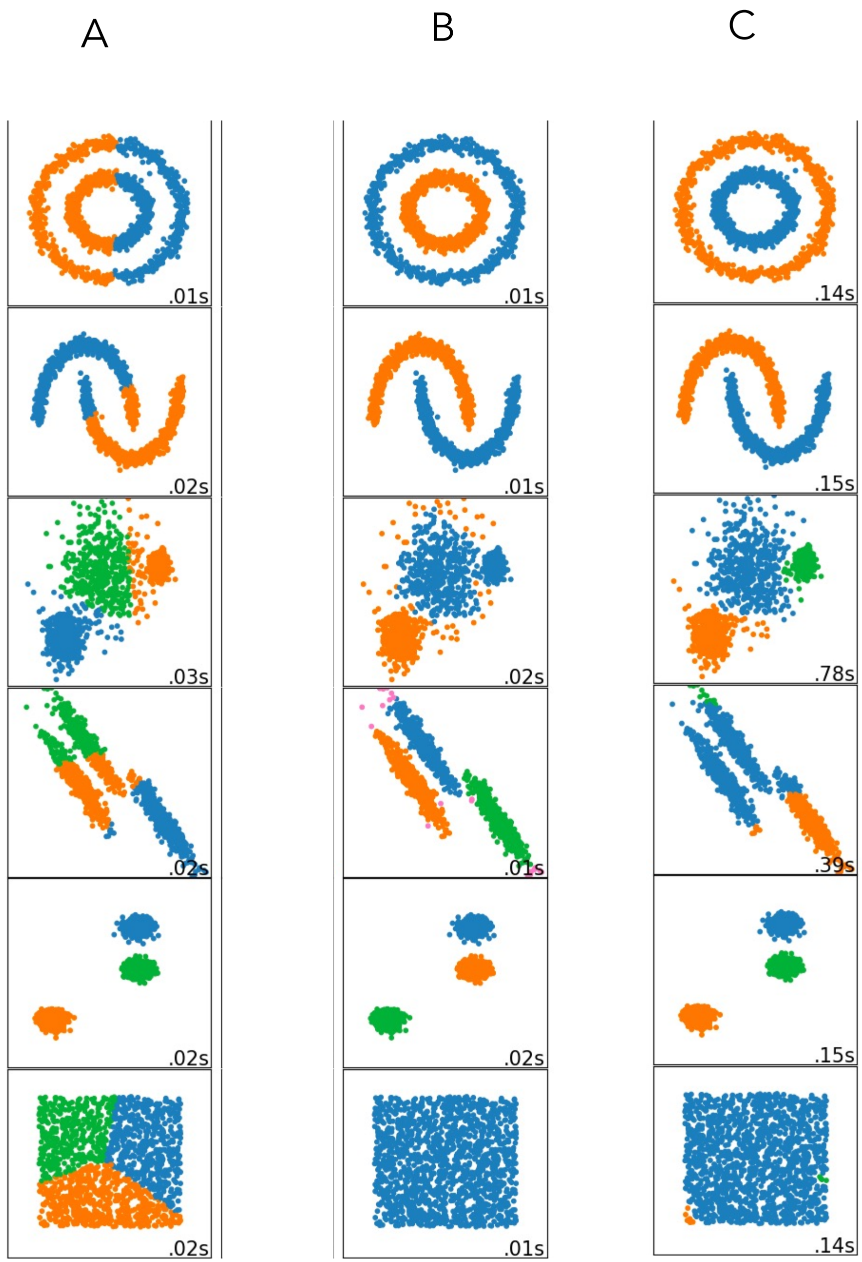
(MinPts=4, Eps=9.92).



(MinPts=4, Eps=9.75)

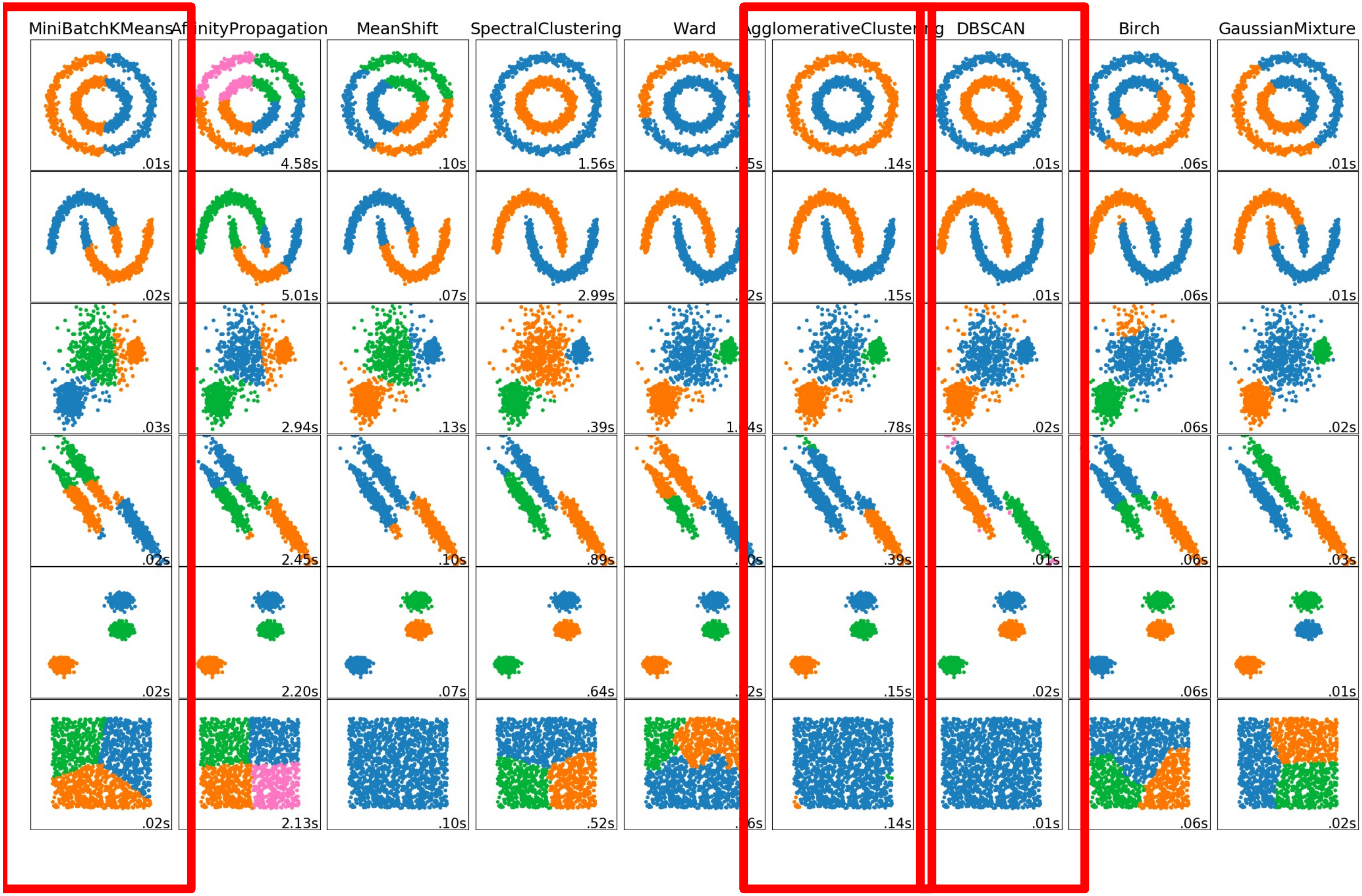
- Cannot handle varying densities
- Sensitive to parameters

WHO WORE IT BEST?



A	KMeans	DBScan	Aggl. clustering
B	Aggl. clustering	KMeans	DBScan
C	KMeans	Aggl. clustering	DBScan

CLUSTERING ANSWER: A WINS



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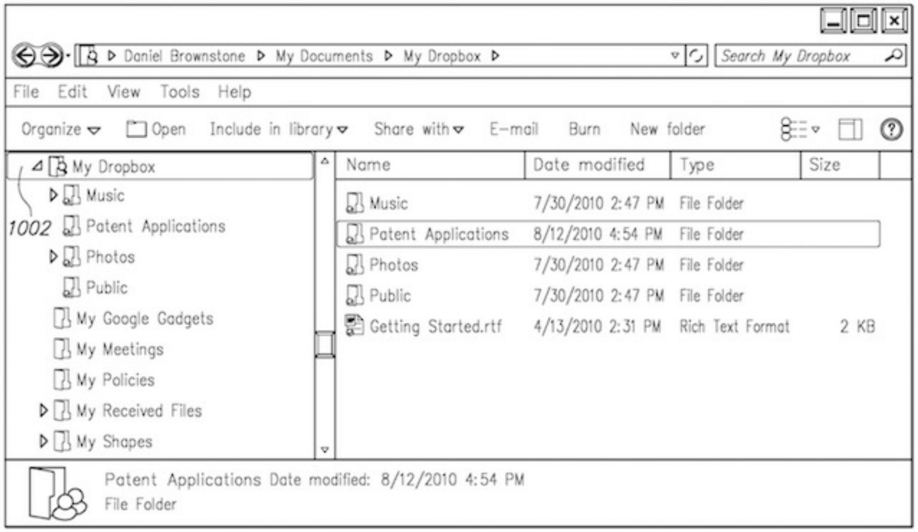
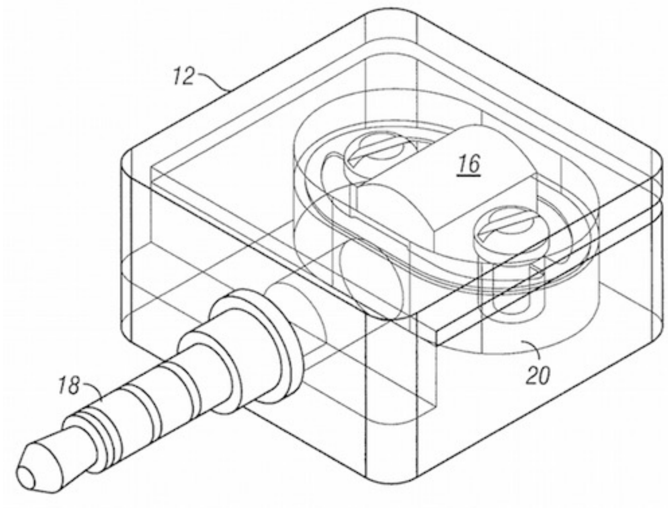
Motivational Example

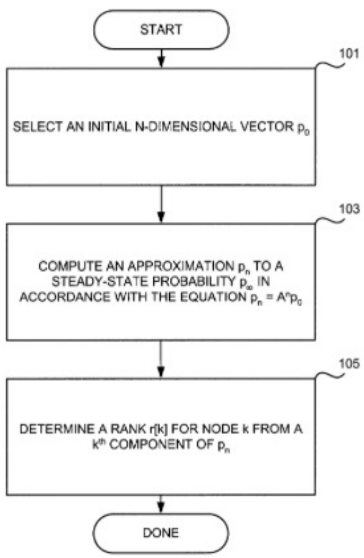
Around **300.000**
US **Patent** Applications
Granted **per Year**



Network folder synchronization (DropBox)



Systems and methods for decoding card s wipe signals (Square)



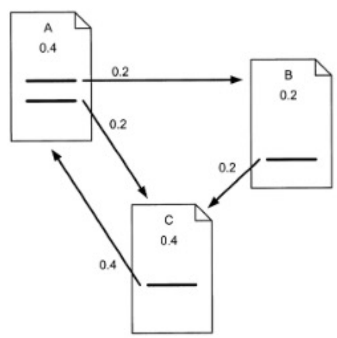


FIG. 2

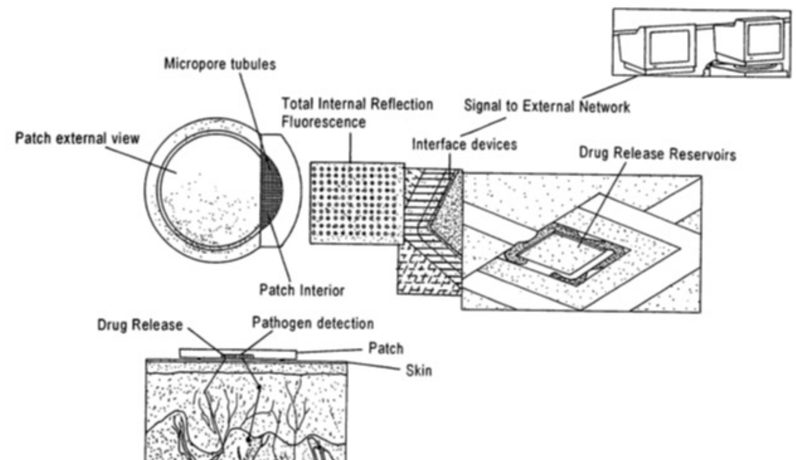
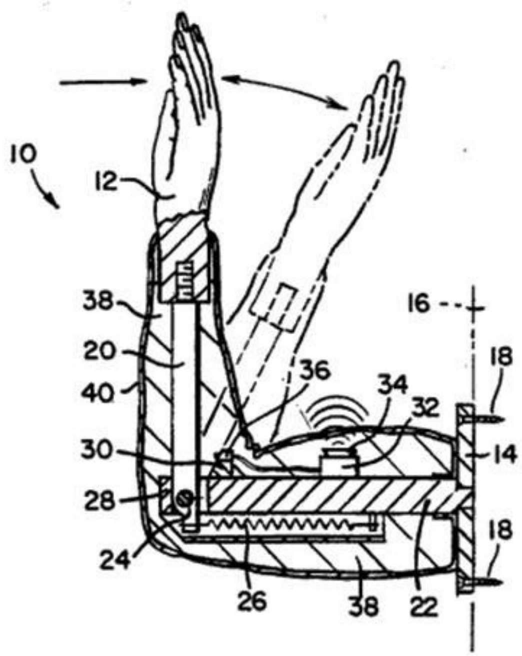


Fig. 2

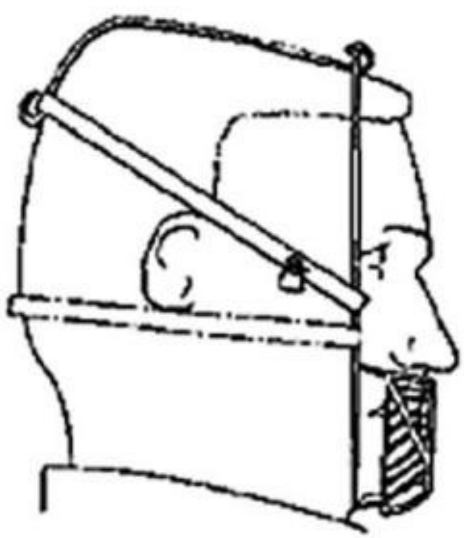
Method for node ranking in a linked database (Google)

Medical device for analyte monitoring and drug delivery (Theranos)

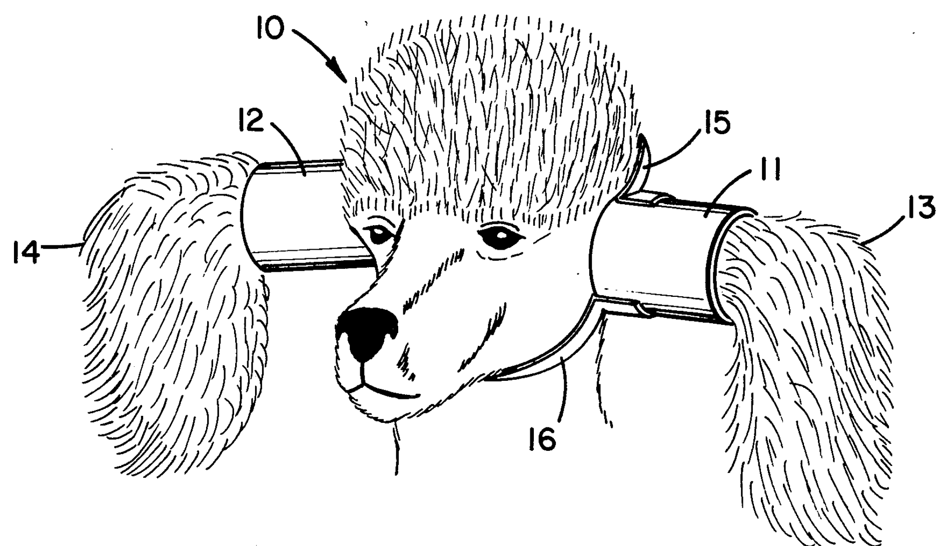


High-Five Machine

Gerbil Shirt



Anti Eating Device



Dog Ear Protection

Note, that I could not actually verify them as real patents, but they could easily be some

Goal:

We want to build a **model** to automatically **classify patents** into useful or bogus?

What do we need?

1. The patent data (easy thanks to Google Patents)
2. A training data set: some pre-labeled patents
3. A model

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How do we get a labeled data set?

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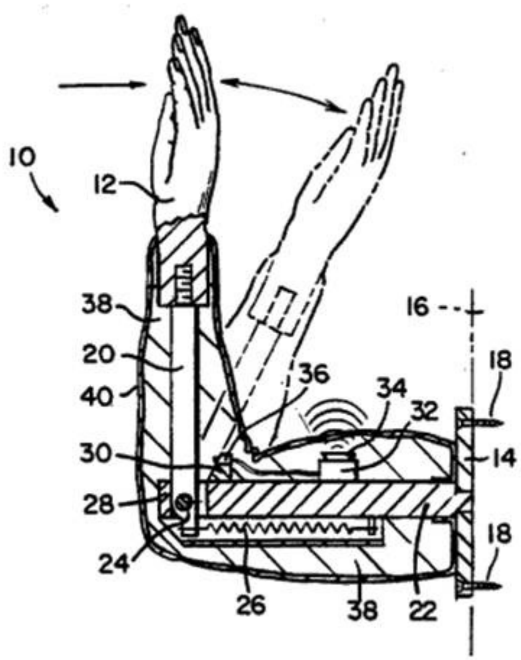
A Crowd Task

Is this Patent Bogus?



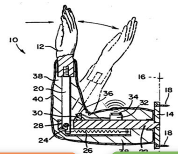
Yes

No

$$l = 1$$

A Crowd Task

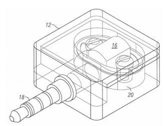
Is this Patent Bogus?



Yes No

o_1

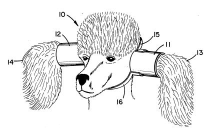
Is this Patent Bogus?



Yes No

o_2

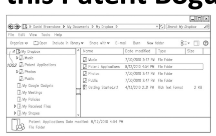
Is this Patent Bogus?



Yes No

o_3

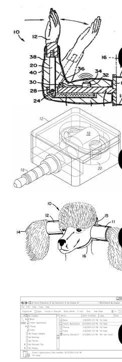
Is this Patent Bogus?



Yes No

o_4

⋮



$$l[n] = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix} \begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ \end{matrix}$$

A Crowd Task

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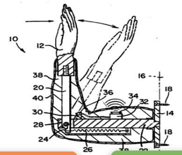

Yes | No

o_1

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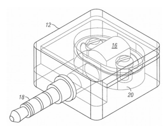
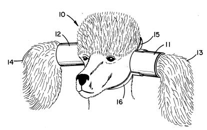


Yes | No

o_2

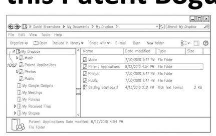
Is this Patent Bogus?



Yes | No

o_3

Is this Patent Bogus?

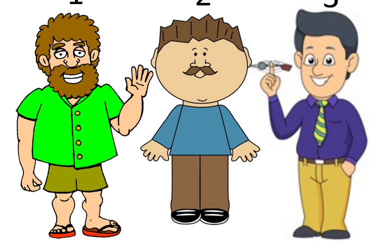


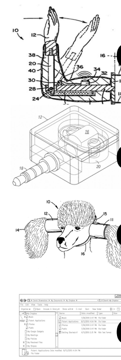
Yes | No

o_4

⋮

k_1 k_2 k_3

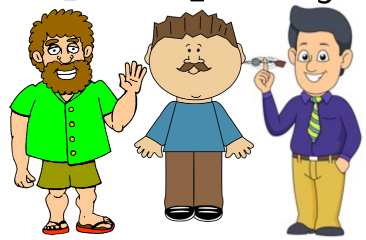


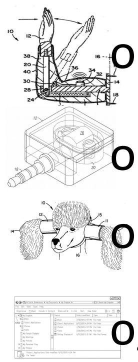


$$l[k][n] = \begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ \end{matrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ \vdots & \vdots & \vdots \end{pmatrix}$$

What should the final labels be?

k_1 k_2 k_3





o_1, o_2, o_3, o_4

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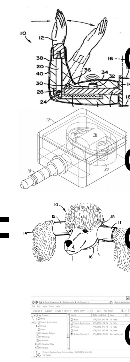


o_1, o_2, o_3, o_4

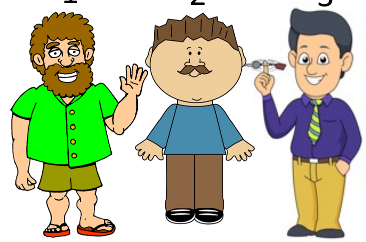
$$T(n) = \begin{pmatrix} ? \\ ? \\ ? \\ ? \\ \vdots \end{pmatrix}$$

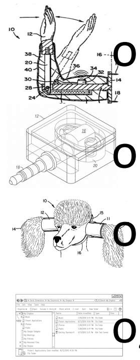
Maximum Likelihood Estimate

- Given some data X= $(x_1, \dots, x_n)$
- Model $\mathcal{L}(\theta, X) = p_\theta(X) = \prod_i^n p_\theta(x_i)$
- **Maximum Likelihood Estimator (MLE)**

$$\hat{\theta} = \operatorname*{argmax}_{\theta \in \Theta} \mathcal{L}(\theta, X)$$

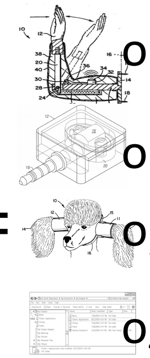
A Maximum Likelihood Estimate (MLE)





$$l[k][n] = \begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ \end{matrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ \vdots & \vdots & \vdots \end{pmatrix}$$

What should the final labels be?



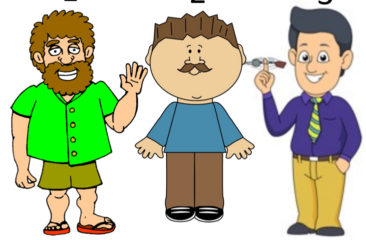
$$T(n) = \begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ \end{matrix} \begin{pmatrix} ? \\ ? \\ ? \\ ? \\ \vdots \end{pmatrix}$$

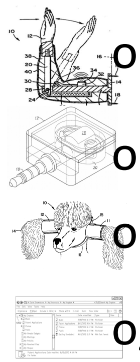
Bogus | Not Bogus

$$\begin{pmatrix} 3/3 & 0/3 \\ 0/3 & 3/3 \\ 2/3 & 1/3 \\ 1/3 & 2/3 \\ \vdots & \vdots \end{pmatrix}$$

A Maximum Likelihood Estimate (MLE)

k_1 k_2 k_3





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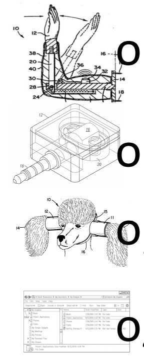

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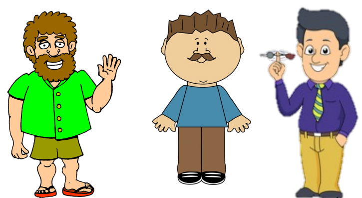
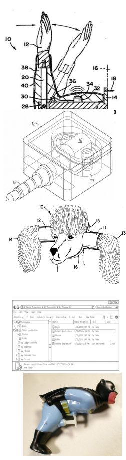
Bogus | Not Bogus

$$\begin{pmatrix} 3/3 & 0/3 \\ 0/3 & 3/3 \\ 2/3 & 1/3 \\ 1/3 & 2/3 \\ \vdots & \vdots \end{pmatrix}$$

So Everything is Good





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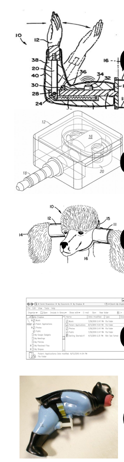
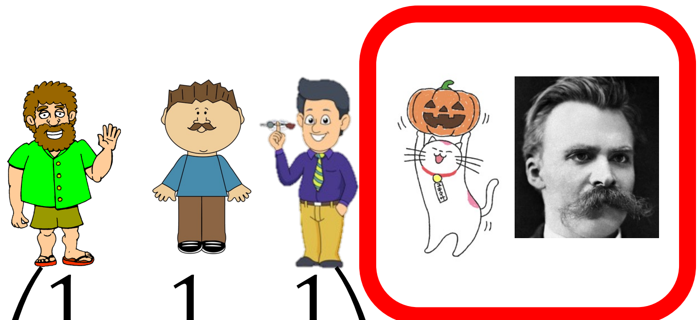
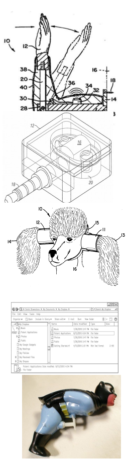


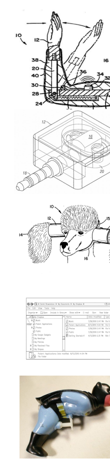
$$T(n) = \begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{matrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

But what happens if we add **Crazy Cat with Pumpkin** and the **Nihilist?**



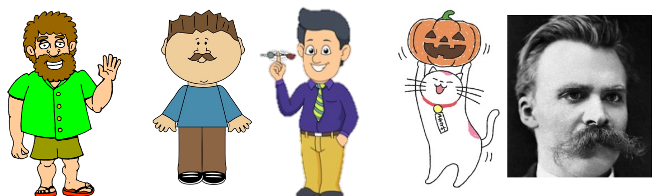


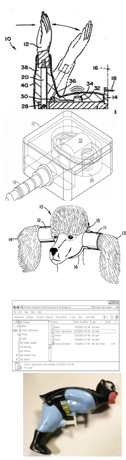
$$\begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{matrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$



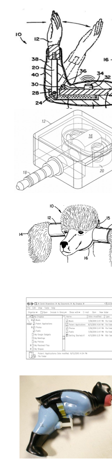
$$T(n) = \begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{matrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

What if the Workers do not have the same Quality?





$$\begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

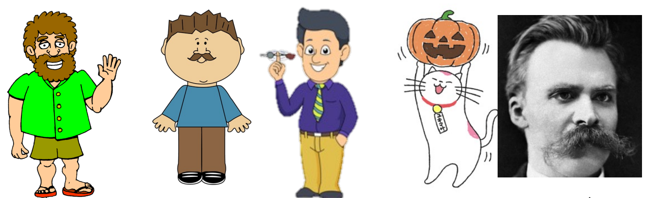


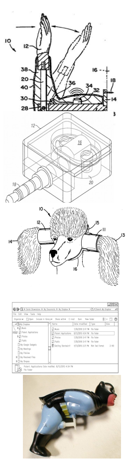
$$T(n) = \begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{matrix} \begin{pmatrix} 1 \\ 0 \\ \mathbf{0} \\ 0 \\ 1 \end{pmatrix}$$

What if the Workers do not have the same Quality?

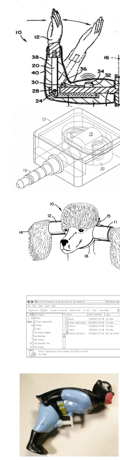
Latent (hidden) Variables

$Z_1 \quad Z_2 \quad Z_3 \quad Z_4 \quad Z_5$





$$\begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$



$$T(n) = \begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{matrix} \begin{pmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{pmatrix}$$

Maximum Likelihood Estimate

- Given some data X= $(x_1, \dots, x_n)$
- Model $\mathcal{L}(\theta, X) = p_\theta(X) = \prod_i^n p_\theta(x_i)$
- **Maximum Likelihood Estimator (MLE)**

$$\hat{\theta} = \underset{\theta \in \Theta}{\operatorname{argmax}} \, \mathcal{L}(\theta, X)$$

Maximum Likelihood Estimate

- Given some data X= $(x_1, \ldots, x_n)$
- Model $\mathcal{L}(\theta, X, Z) = p_\theta(X, Z) = \prod_{i}^{n} p_\theta(x_i, z)$
- **Maximum Likelihood Estimator (MLE)**

$$\hat{\theta} = \underset{\theta}{\operatorname{argmax}} \mathcal{L}(\theta, X) = \sum_{z} p_\theta(X, Z)$$

  - Z has been marginalized
  - Hard to compute

Expectation Maximization Algorithm

```
Initialize θ ∈ Θ
For t = 0,1,2,…
     E-Step:   Calculate the expected value
               of the log likelihood
               function, with respect to the
               conditional distribution of Z
               given X under the current
               estimate of the parameters θ_t:
```

$$Q(Q|\theta_t) = E_{Z|X,\theta_t}[\log \mathcal{L}(\theta, X, Z)]$$

```
     M-Step:   Find the parameter that
               maximizes this quantity
```

$$\theta_{t+1} = \underset{\theta}{\operatorname{argmax}}\, Q(Q|\theta_t)$$

EM – In our Example

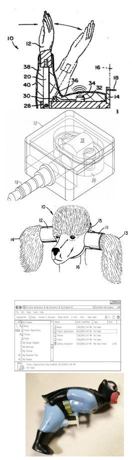
Initialize θ_0

For t = 0,1,2,…

E-Step: Calculate the expected labels (e.g., bogus or not-bogus) given θ_t

M-Step: Given the estimated label, optimize θ and set it to θ_{t+1}

EM Algorithm - Example

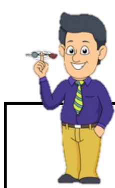


o_1	1	1	1	0	0
o_2	0	0	0	1	1
o_3	0	1	1	0	0
o_4	0	0	0	0	1
o_5	0	1	1	1	0

Guess

True

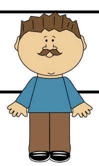
	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1



Guess

True

	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1



Guess

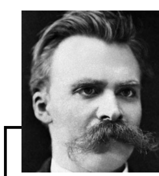
True

	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

Guess

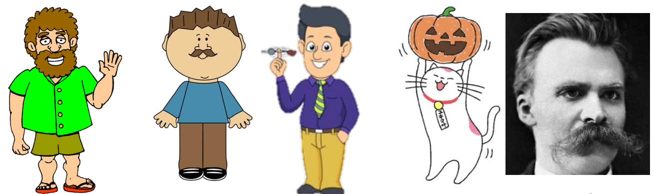
True

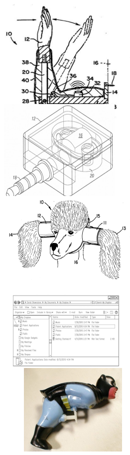
	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1



	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

EM Algorithm - Example





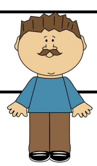
$$\begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Bogus / Not Bogus

$$\begin{pmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \\ 0.4 & 0.6 \\ 0.2 & 0.8 \\ 0.6 & 0.4 \end{pmatrix}$$

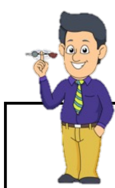
Guess

True	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1



Guess

True	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

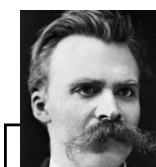


Guess

True	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

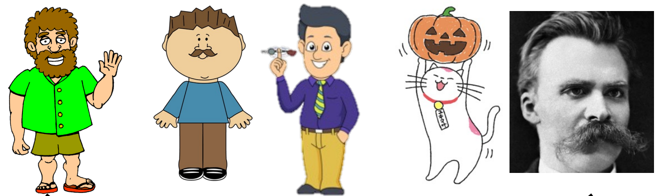
Guess

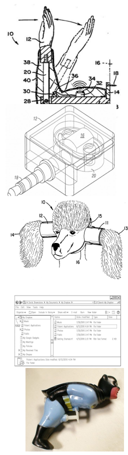
True	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1



	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

EM Algorithm - Example





$$
\begin{array}{c}o_1\\o_2\\o_3\\o_4\\o_5\end{array}
\begin{pmatrix}
1 & 1 & 1 & 0 & 0\\
0 & 0 & 0 & 1 & 1\\
0 & 1 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 1\\
0 & 1 & 1 & 1 & 0
\end{pmatrix}
$$

Bogus | Not Bogus

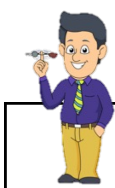
$$
\begin{pmatrix}
0.6 & 0.4\\
0.4 & 0.6\\
0.4 & 0.6\\
0.2 & 0.8\\
0.6 & 0.4
\end{pmatrix}
$$

"Correct" Labels

$$
\begin{pmatrix}
1\\0\\0\\0\\1
\end{pmatrix}
$$

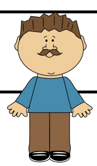
Guess

True	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1



Guess

True	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

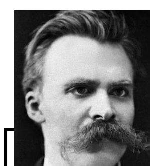


Guess

True	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

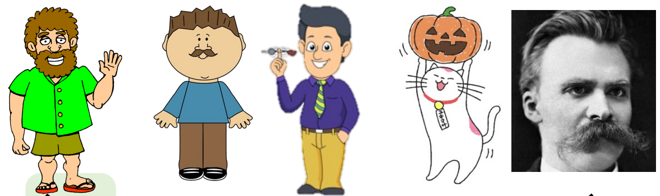
Guess

True	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

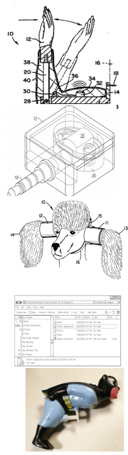


	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

EM Algorithm - Example



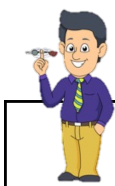
"Correct" Labels



$$\begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

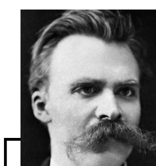
Guess

True		Bogus	!Bogus
	Bogus	?	?
	!Bogus	?	?

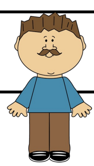


Guess

True		Bogus	!Bogus
	Bogus	?	?
	!Bogus	?	?



	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?



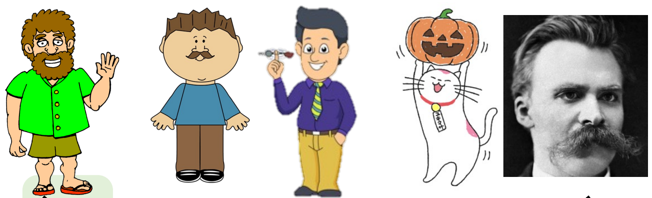
Guess

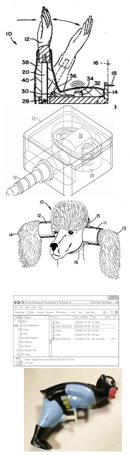
True		Bogus	!Bogus
	Bogus	?	?
	!Bogus	?	?

Guess

True		Bogus	!Bogus
	Bogus	?	?
	!Bogus	?	?

EM Algorithm - Example





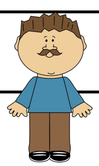
$$\begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"Correct" Labels

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

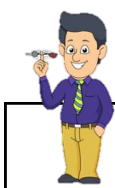
Guess

True		Bogus	!Bogus
	Bogus	1	0.25
	!Bogus	0	0.75



Guess

True		Bogus	!Bogus
	Bogus	?	?
	!Bogus	?	?

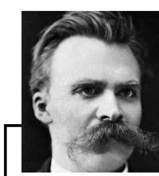


Guess

True		Bogus	!Bogus
	Bogus	?	?
	!Bogus	?	?

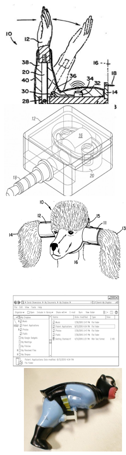
Guess

True		Bogus	!Bogus
	Bogus	?	?
	!Bogus	?	?



	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

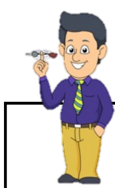
EM Algorithm - Example



$$
\begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{matrix}
\begin{pmatrix}
1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 0
\end{pmatrix}
\qquad
\text{“Correct” Labels} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}
$$

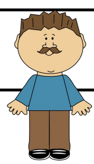
Guess

True	Bogus	!Bogus
Bogus	1	0.25
!Bogus	0	0.75



Guess

True	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

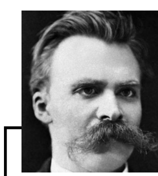


Guess

True	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

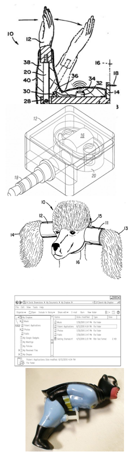
Guess

True	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?



	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

EM Algorithm - Example

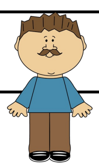


$$
\begin{array}{c} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{array}
\begin{pmatrix}
1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 0
\end{pmatrix}
\qquad
\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}
$$

“Correct” Labels

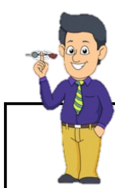
Guess

True	Bogus	!Bogus
Bogus	1	0.25
!Bogus	0	0.75



Guess

True	Bogus	!Bogus
Bogus	0.66	0
!Bogus	0.33	1

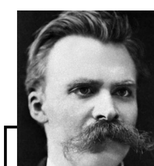


Guess

True	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

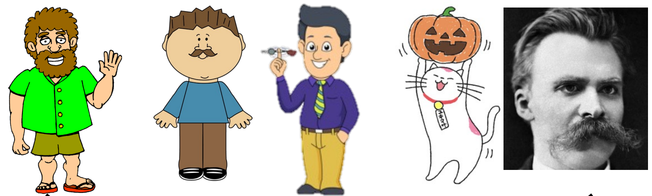
Guess

True	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

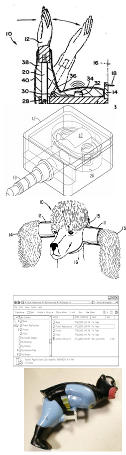


	Bogus	!Bogus
Bogus	?	?
!Bogus	?	?

EM Algorithm - Example



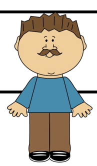
"Correct" Labels



$$\begin{array}{l} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{array} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

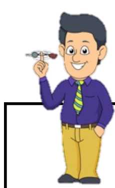
Guess

True	Bogus	!Bogus
Bogus	1	0.25
!Bogus	0	.75



Guess

True	Bogus	!Bogus
Bogus	0.66	0
!Bogus	0.33	1

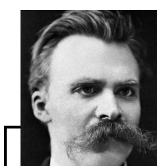


Guess

True	Bogus	!Bogus
Bogus	.66	0
!Bogus	.33	1

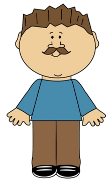
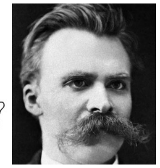
Guess

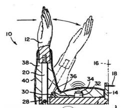
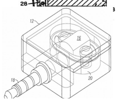
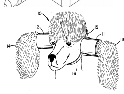
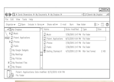
True	Bogus	!Bogus
Bogus	.5	0.33
!Bogus	.5	0.66



	Bogus	!Bogus
Bogus	0	0.66
!Bogus	1	0.33

EM Algorithm - Example

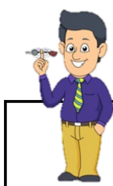
	o_1
	o_2
	o_3
	o_4
	o_5

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{pmatrix}$$

“Correct” Labels

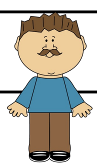
Guess

True	Bogus	!Bogus
Bogus	1	0.25
!Bogus	0	.75



Guess

True	Bogus	!Bogus
Bogus	.66	0
!Bogus	.33	1

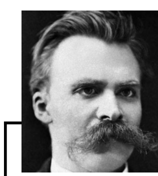


Guess

True	Bogus	!Bogus
Bogus	0.66	0
!Bogus	0.33	1

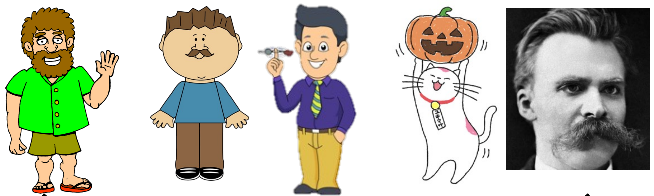
Guess

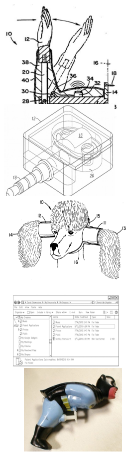
True	Bogus	!Bogus
Bogus	.5	0.33
!Bogus	.5	0.66



	Bogus	!Bogus
Bogus	0	0.66
!Bogus	1	0.33

EM Algorithm - Example





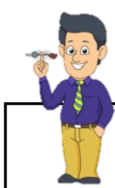
$$\begin{matrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Bogus   Not Bogus

$$\begin{pmatrix} 1 + .66 + .66 + .33 + .66 & 0 + .33 + .33 + .66 + .33 \\ 0.25 + 0 + 0 + .5 + 0 & .75 + 1 + 1 + 0.5 + 1 \\ 0.25 + .66 + .66 + 0.33 + .66 & .75 + .33 + .33 + 0.66 + .33 \\ 0.25 + 0 + 0 + .33 + 0 & .75 + 1 + 1 + .66 + 1 \\ .25 + .66 + .66 + .5 + .66 & .75 + .33 + .33 + .5 + .33 \end{pmatrix}$$

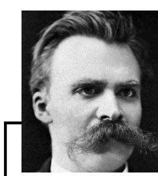
Guess

True	Bogus	!Bogus
Bogus	1	0.25
!Bogus	0	.75

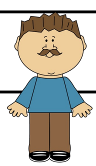


Guess

True	Bogus	!Bogus
Bogus	.66	0
!Bogus	.33	1



	Bogus	!Bogus
Bogus	0	0.66
!Bogus	1	0.33



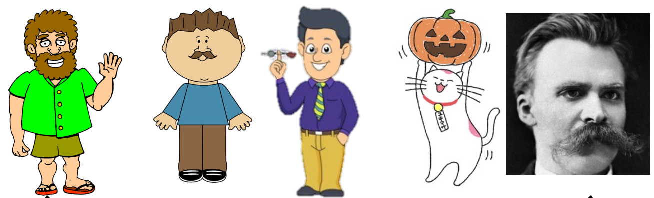
Guess

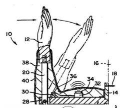
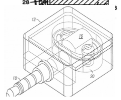
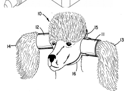
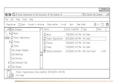
True	Bogus	!Bogus
Bogus	0.66	0
!Bogus	0.33	1

Guess

True	Bogus	!Bogus
Bogus	.5	0.33
!Bogus	.5	0.66

EM Algorithm - Example



	o_1	1	1	1	0	0
	o_2	0	0	0	1	1
	o_3	0	1	1	0	0
	o_4	0	0	0	0	1
	o_5	0	1	1	1	0

Bogus	Not Bogus
$1 + .66 + .66 + .33 + .66$	$0 + .33 + .33 + .66 + .33$
$0.25 + 0 + 0 + .5 + 0$	$.75 + 1 + 1 + 0.5 + 1$
$0.25 + .66 + .66 + 0.33 + .66$	$.75 + .33 + .33 + 0.66 + .33$
$0.25 + 0 + 0 + .33 + 0$	$.75 + 1 + 1 + .66 + 1$
$.25 + .66 + .66 + .5 + .66$	$.75 + .33 + .33 + .5 + .33$

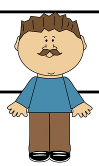
Guess

True

	Bogus	!Bogus
Bogus	1	0.25
!Bogus	0	.75

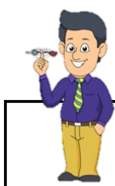
Guess

True

	Bogus	!Bogus
Bogus	0.66	0
!Bogus	0.33	1

Guess

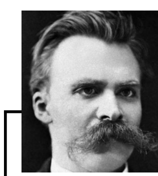
True

	Bogus	!Bogus
Bogus	.66	0
!Bogus	.33	1

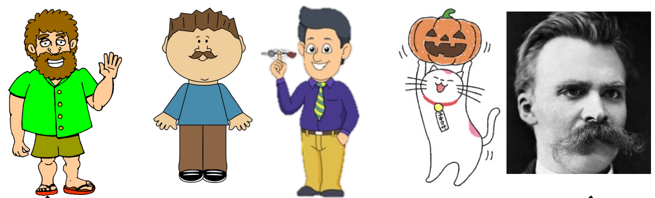
Guess

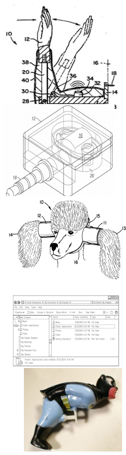
True

	Bogus	!Bogus
Bogus	.5	0.33
!Bogus	.5	0.66

	Bogus	!Bogus
Bogus	0	0.66
!Bogus	1	0.33

EM Algorithm - Example



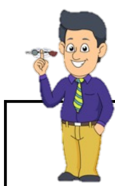


o_1	1	1	1	0	0
o_2	0	0	0	1	1
o_3	0	1	1	0	0
o_4	0	0	0	0	1
o_5	0	1	1	1	0

Bogus	Not Bogus
1 + .66 + .66 + .33 + .66	0 + .33 + .33 + .66 + .33
0.25 + 0 + 0 + .5 + 0	.75 + 1 + 1 + 0.5 + 1
0.25 + .66 + .66 + 0.33 + .66	.75 + .33 + .33 + 0.66 + .33
0.25 + 0 + 0 + .33 + 0	.75 + 1 + 1 + .66 + 1
.25 + .66 + .66 + .5 + .66	.75 + .33 + .33 + .5 + .33

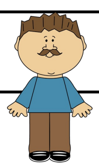
Guess

True	Bogus	!Bogus
Bogus	1	0.25
!Bogus	0	.75



Guess

True	Bogus	!Bogus
Bogus	.66	0
!Bogus	.33	1

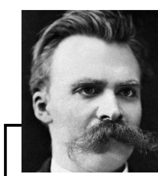


Guess

True	Bogus	!Bogus
Bogus	0.66	0
!Bogus	0.33	1

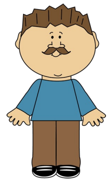
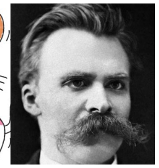
Guess

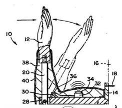
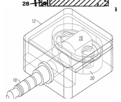
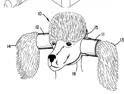
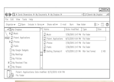
True	Bogus	!Bogus
Bogus	.5	0.33
!Bogus	.5	0.66



	Bogus	!Bogus
Bogus	0	0.66
!Bogus	1	0.33

EM Algorithm - Example

 o_1
 o_2
 o_3
 o_4
 o_5

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Bogus, Not Bogus

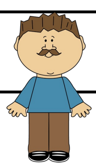
$$\begin{pmatrix} 0.66 & .33 \\ 0.15 & .85 \\ 0.52 & 0.48 \\ .12 & 0.88 \\ 0.55 & 0.45 \end{pmatrix}$$

"Correct" Labels

$$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

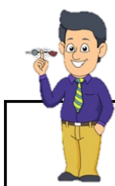
Guess

True		Bogus	!Bogus
	Bogus	1	0.25
	!Bogus	0	.75



Guess

True		Bogus	!Bogus
	Bogus	0.66	0
	!Bogus	0.33	1

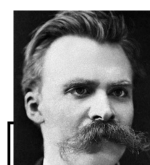


Guess

True		Bogus	!Bogus
	Bogus	.66	0
	!Bogus	.33	1

Guess

True		Bogus	!Bogus
	Bogus	.5	0.33
	!Bogus	.5	0.66



	Bogus	!Bogus
Bogus	0	0.66
!Bogus	1	0.33

Dawid and Skene EM Algorithm [1]

Input: Labels $l[k][n]$ from worker (k) to object o_n,

Output: Confusion matrix $\pi_{ij}^{(k)}$ for each worker (k), Correct labels $T(o_n)$ for each object o_n, Class priors $Pr\{C\}$ for each class C

1 Initialize error rates $\pi_{ij}^{(k)}$ for each worker (k) (e.g., assume each worker is perfect);

2 Initialize correct label for each object $T(o_n)$ (e.g., using majority vote);

3 **while** *not converged* **do**

4   Estimate the correct label $T(o_n)$ for each object, using the labels $l[\cdot][n]$ assigned to o_n by workers, weighting the votes using the error rates $\pi_{ij}^{(k)}$;

5   Estimate the error rates $\pi_{ij}^{(k)}$, for each worker (k), using the correct labels $T(o_n)$ and the assigned labels $l[k][n]$;

6   Estimate the class priors $Pr\{C\}$, for each class C;

7 **end**

8 **return** *Estimated error rates* $\pi_{ij}^{(k)}$, *Estimated correct labels* $T(o_n)$, *Estimated class priors* $Pr\{C\}$

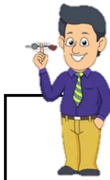
[1] Panos Ipeirotis, Foster Provost, Jing Wang: **Quality management on Amazon Mechanical Turk.** Proceedings of the ACM SIGKDD Workshop on Human Computation, 2010

[2] Dawid, A. P., and Skene, A. M. **Maximum likelihood estimation of observer error-rates using the EM algorithm**. Applied Statistics 28, 1 (Sept. 1979), 20–28.

Confusion Matrices in the 2nd iteration

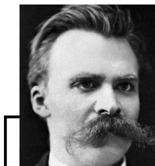
True

	Bogus	!Bogus
Bogus	1	0.5
!Bogus	0	.5

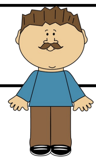


True

	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1



	Bogus	!Bogus
Bogus	0	1
!Bogus	1	0



True

	Bogus	!Bogus
Bogus	1	0
!Bogus	0	1

True

	Bogus	!Bogus
Bogus	.5	0.66
!Bogus	.5	0.33

Which worker is the worst?

EM-Algorithm: Many other applications

```
Initialize θ ∈ Θ
For t = 0,1,2,…
    E-Step:   Calculate the expected value
              of the log likelihood
              function, with respect to the
              conditional distribution of Z
              given X under the current
              estimate of the parameters θ_t:
```

$$Q\left(Q \middle| \theta_t\right) = E_{Z|X,\theta_t}[\log \mathcal{L}(\theta, X, Z)]$$

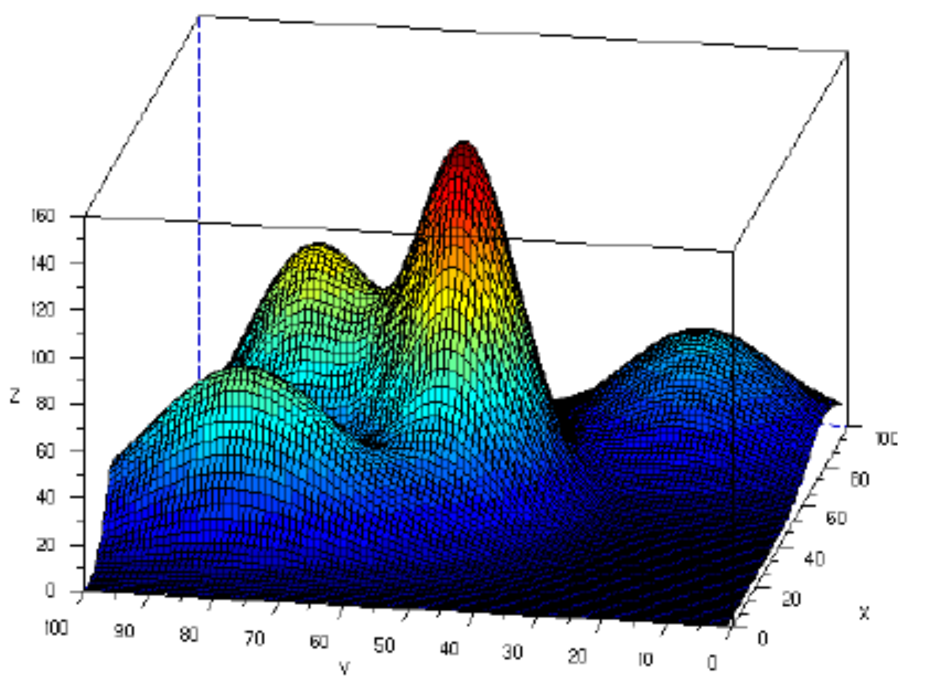
```
    M-Step:   Find the parameter that
              maximizes this quantity
```

$$\theta_{t+1} = \underset{\theta}{\operatorname{argmax}}\, Q\left(Q \middle| \theta_t\right)$$

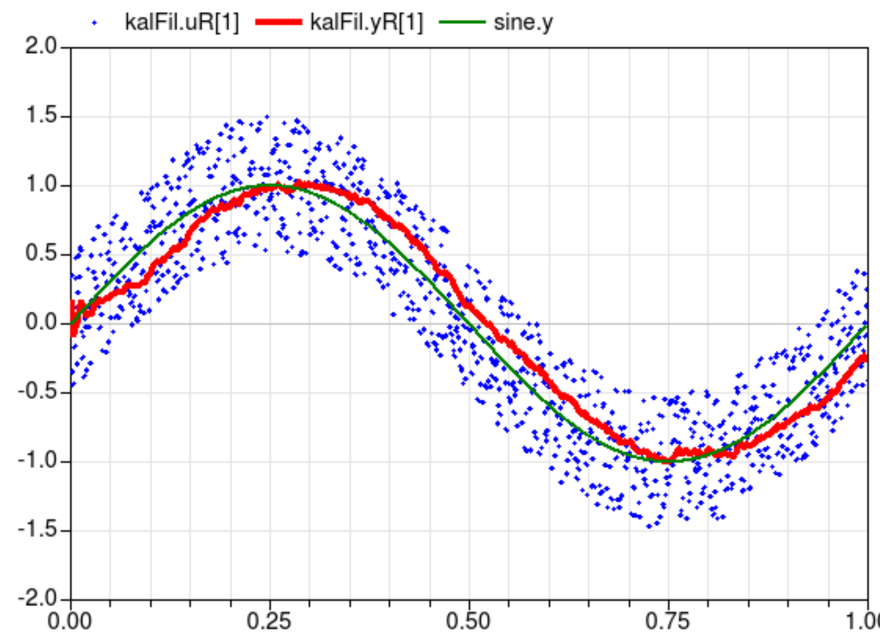
EM-Algorithm: Many other applications

Initialize $\theta \in \Theta$

Gaussian Mixture Models (GMM)



Kalman filter



maximizes this quantity

$$\theta_{t+1} = \underset{\theta}{\operatorname{argmax}}\, Q(Q|\theta_t)$$

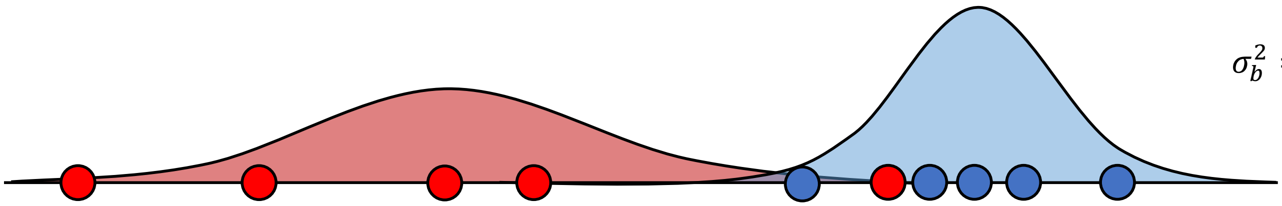
EM-Algorithm: Gaussian Mixture Models

Unknown distributions parameters, known data point labels

- K=2 Gaussians with unknown **μ**, **σ**
- Assume you know if data point comes from the red or blue distribution.
- Estimating the distribution parameters is trivial

$$\mu_b = \frac{x_1 + x_2 + \cdots + x_n}{n_b}$$

$$\sigma_b^2 = \frac{(x_1 - \mu_b)^2 + (x_2 - \mu_b)^2 + \cdots + n - \mu_b)^2}{n_b}$$



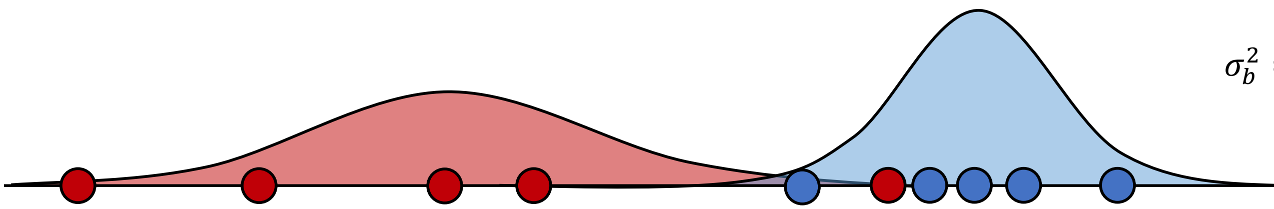
EM-Algorithm: Gaussian Mixture Models

Unknown distributions parameters, known data point labels

- K=2 Gaussians with unknown **μ**, **σ**
- Assume you know if data point comes from the red or blue distribution.
- Estimating the distribution parameters is trivial

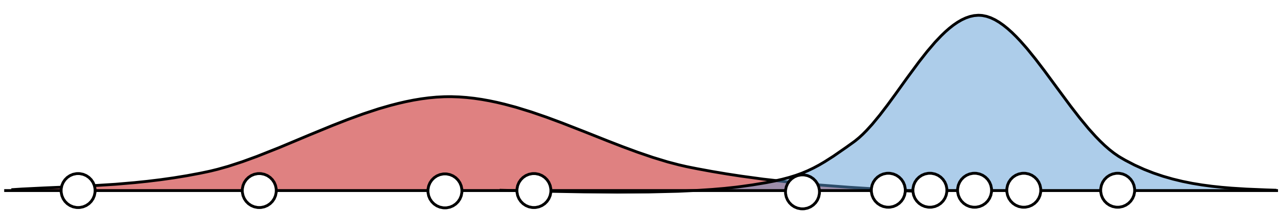
$$\mu_b = \frac{x_1 + x_2 + \cdots + x_n}{n_b}$$

$$\sigma_b^2 = \frac{(x_1 - \mu_b)^2 + (x_2 - \mu_b)^2 + \cdots + (x_n - \mu_b)^2}{n_b}$$



Known distributions parameters, unknown data point labels

- We can guess whether the point is more likely from the blue or red distribution



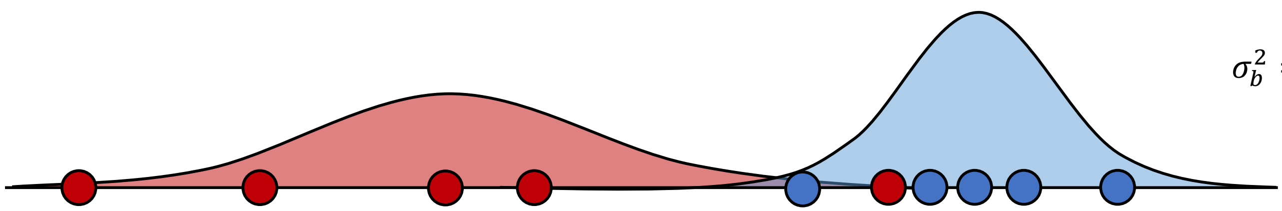
EM-Algorithm: Gaussian Mixture Models

Unknown distributions parameters, known data point labels

- K=2 Gaussians with unknown **μ**, **σ**
- Assume you know if data point comes from the red or blue distribution.
- Estimating the distribution parameters is trivial

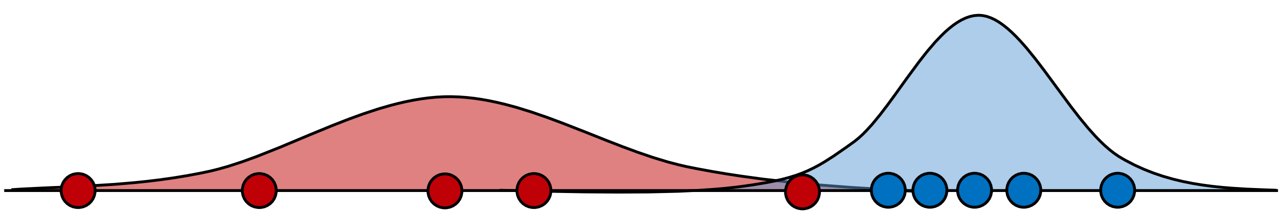
$$\mu_b = \frac{x_1 + x_2 + \cdots + x_n}{n_b}$$

$$\sigma_b^2 = \frac{(x_1 - \mu_b)^2 + (x_2 - \mu_b)^2 + \cdots + n - \mu_b)^2}{n_b}$$



Known distributions parameters, unknown data point labels

- We can guess weather the point is more likely from the blue or red distribution



$$P(b|x_1) = \frac{P(x_1|b)P(b)}{P(x_1|b)P(b) + P(x_1|r)P(r)}$$

$$P(x_1|b) = \frac{1}{\sqrt{2\pi\sigma_b^2}} \exp\left(-\frac{(x-\mu_b)^2}{2\sigma_b^2}\right)$$

EM-Algorithm: Gaussian Mixture Models

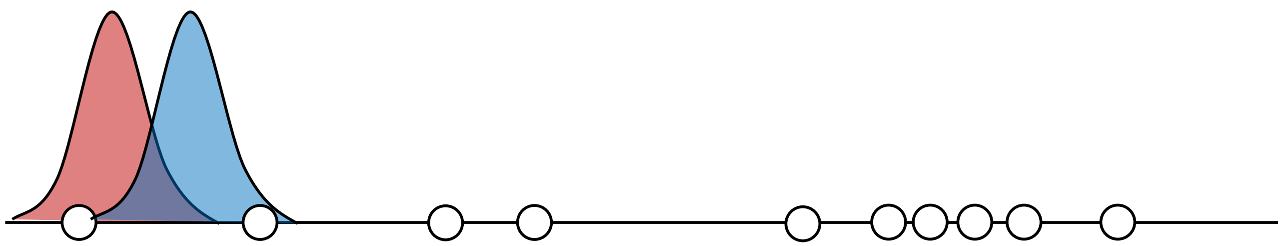
Chicken and egg problem

- Need(μ_r, σ_r^2) and (μ_b, σ_b^2) to guess source of points
- Need to know the source to estimate (μ_r, σ_r^2) and (μ_b, σ_b^2)

EM algorithm

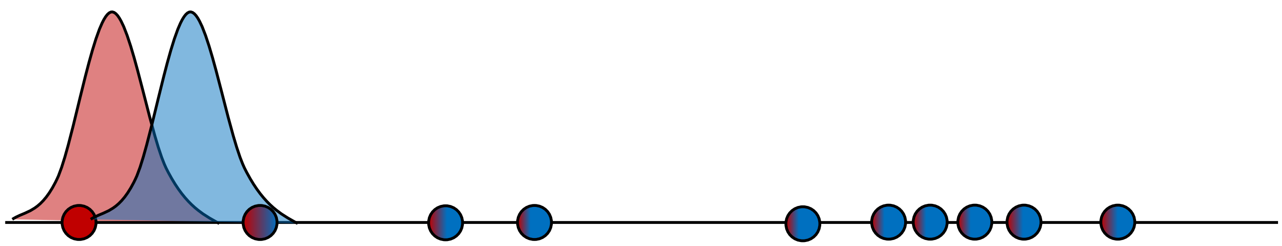
- Start with two randomly placed Gaussians (μ_r, σ_r^2) and (μ_b, σ_b^2)
- E-step: For each point: $P(b|x_1)$ = does it look it came from blue (red)
- M-Step: adjust (μ_r, σ_r^2) and (μ_b, σ_b^2) to fit points assigned to them
- Iterate until convergence

EM-Algorithm: Gaussian Mixture Models



- Start with two randomly placed Gaussians (μ_r, σ_r^2) and (μ_b, σ_b^2)

EM-Algorithm: Gaussian Mixture Models



- Start with two randomly placed Gaussians (μ_r, σ_r^2) and (μ_b, σ_b^2)
- E-step: For each point: $P(b|x_1)$ = does it look it came from blue (red)

E-Step:

$$P(x_1|b) = \frac{1}{\sqrt{2\pi\sigma_b^2}} \exp\left(-\frac{(x-\mu_b)^2}{2\sigma_b^2}\right)$$

Note: we could estimate priors P(b) and P(r), but often left at equal chance

$$b_i = P(b|x_i) = \frac{P(x_i|b)P(b)}{P(x_i|b)P(b) + P(x_i|r)P(r)}$$

$$r_i = 1 - b_i$$

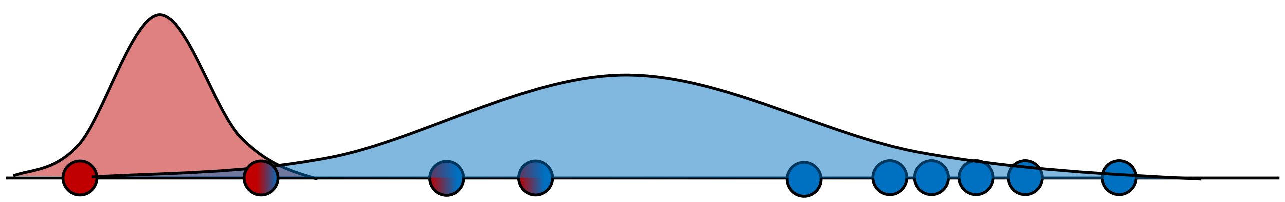
- M-Step: adjust (μ_r, σ_r^2) and (μ_b, σ_b^2) to fit points assigned to them

M-Step:

$$\mu_b = \frac{b_1 x_1 + b_2 x_2 + \cdots + b_n x_n}{b_1 + b_2 + \cdots + b_n}$$

$$\sigma_b^2 = \frac{b_1(x_1 - \mu_b)^2 + b_2(x_2 - \mu_b)^2 + \cdots + b_n(n - \mu_b)^2}{n_b}$$

EM-Algorithm: Gaussian Mixture Models



- Start with two randomly placed Gaussians (μ_r, σ_r^2) and (μ_b, σ_b^2)
- E-step: For each point: $P(b|x_1)$ = does it look it came from blue (red)

E-Step:

$$P(x_1|b) = \frac{1}{\sqrt{2\pi\sigma_b^2}} \exp\left(-\frac{(x-\mu_b)^2}{2\sigma_b^2}\right)$$

$$b_i = P(b|x_i) = \frac{P(x_i|b)P(b)}{P(x_i|b)P(b) + P(x_i|r)P(r)}$$

$$r_i = 1 - b_i$$

Note: we could estimate priors P(b) and P(r), but often left at equal chance

- M-Step: adjust (μ_r, σ_r^2) and (μ_b, σ_b^2) to fit points assigned to them

M-Step:

$$\mu_b = \frac{b_1 x_1 + b_2 x_2 + \cdots + b_n x_n}{b_1 + b_2 + \cdots + b_n}$$

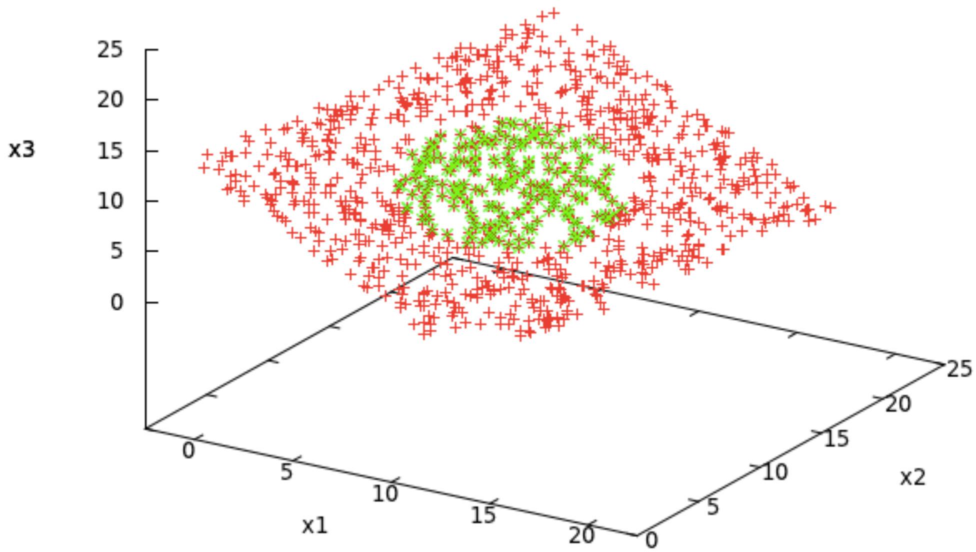
$$\sigma_b^2 = \frac{b_1(x_1 - \mu_b)^2 + b_2(x_2 - \mu_b)^2 + \cdots + b_n(n - \mu_b)^2}{n_b}$$

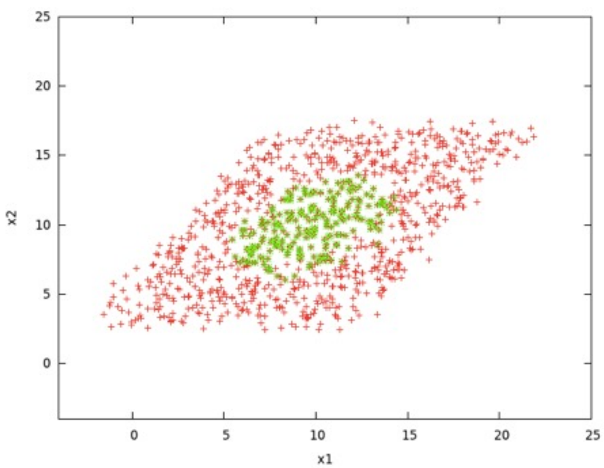
In what way is the algorithm similar to k-means and in what ways is it different?

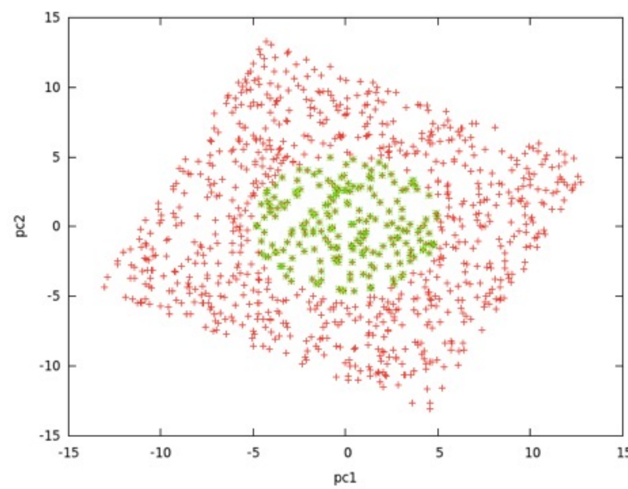
Machine Learning Problems

	Supervised Learning	Unsupervised Learning
Discrete	classification or categorization	clustering
Continuous	regression	dimensionality reduction

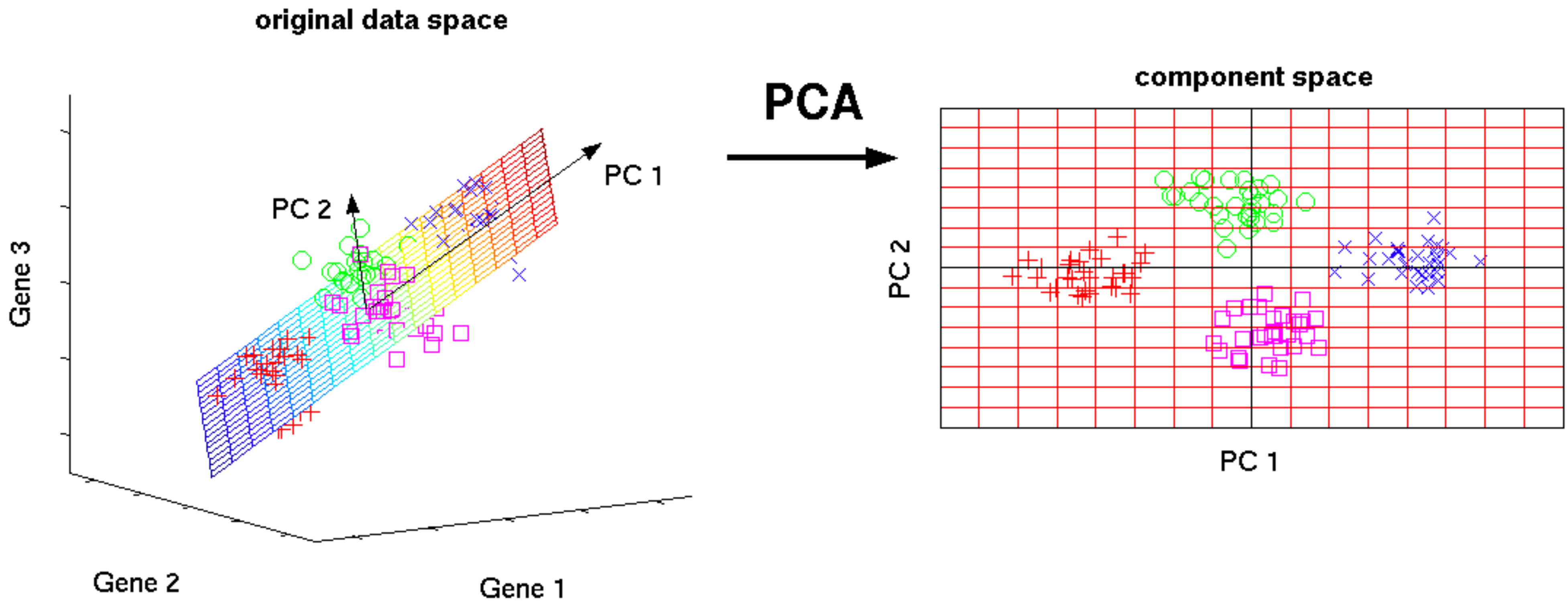
PRINCIPAL COMPONENT ANALYSIS (PCA)



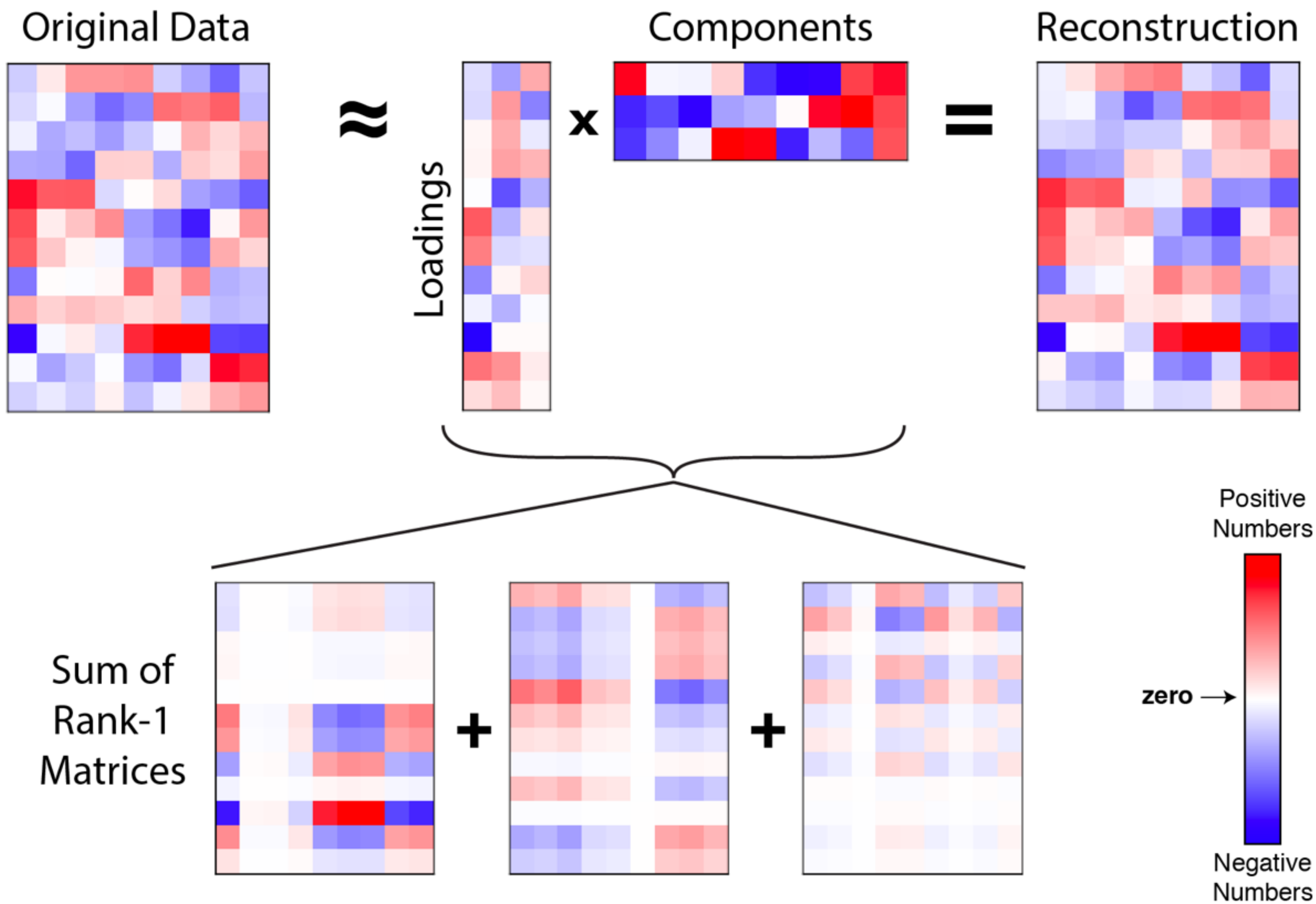


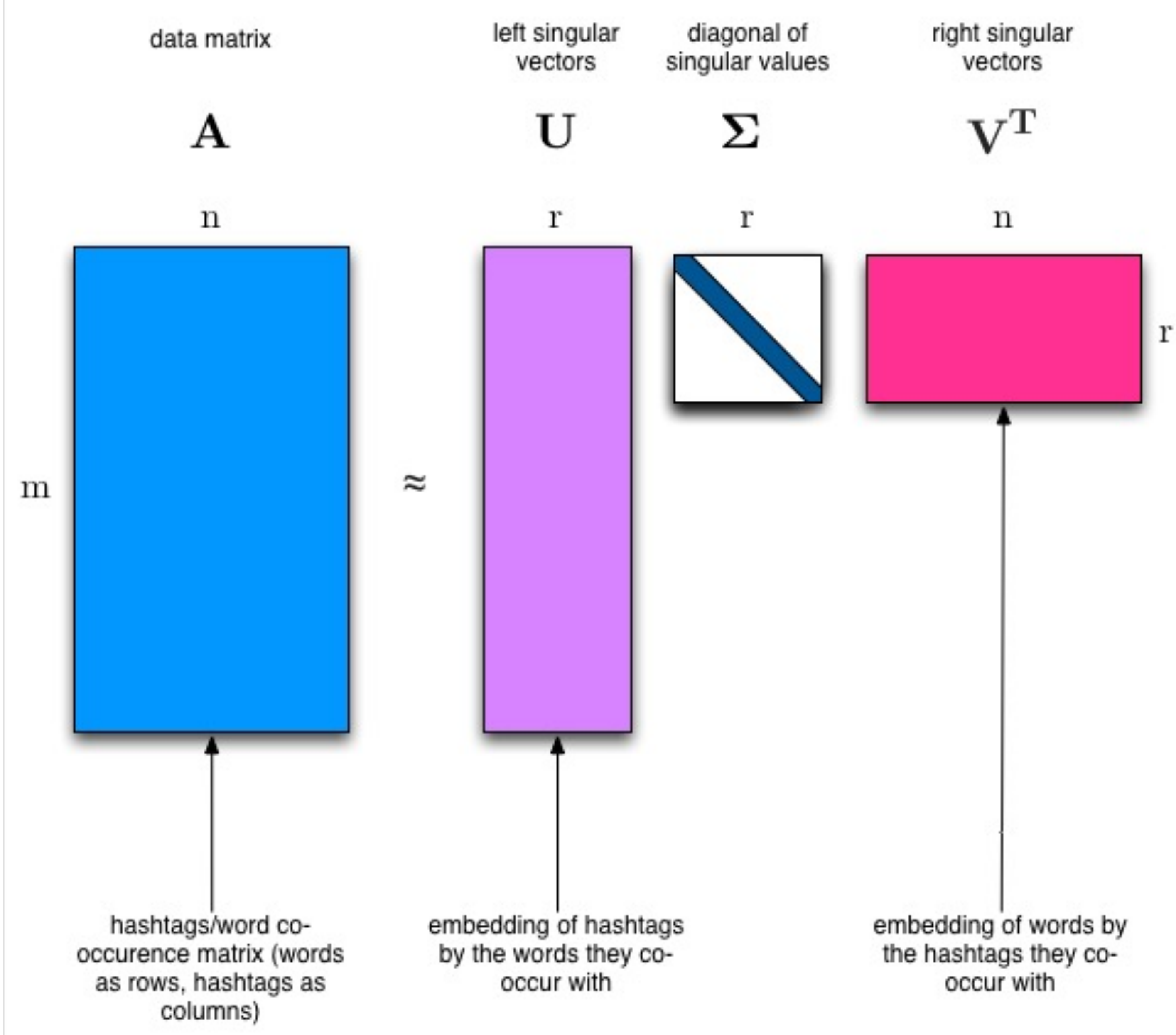


PCA


original data space
PC 1
PC 2
Gene 3
Gene 2
Gene 1
PCA
component space
PC 2
PC 1

PCA Intuition




data matrix
left singular vectors
diagonal of singular values
right singular vectors
A
U
Σ
V^T
n
r
r
n
m
≈
r
hashtags/word co-occurence matrix (words as rows, hashtags as columns)
embedding of hashtags by the words they co-occur with
embedding of words by the hashtags they co-occur with